BAAO
British Astronomy and Astrophysics Olympiad

British Astronomy and Astrophysics Olympiad 2020-2021

Astronomy & Astrophysics Competition Paper

Saturday 27th February 2021

Instructions

Time: 3 hours ($\sim$ 45 minutes for Q1, $\sim$ 65 minutes for Q2 and $\sim$ 70 minutes for Q3).

Questions: All three questions should be attempted. Each question contains independent parts so that later parts can be attempted even if earlier parts are incomplete.

Solutions: Answers and calculations are to be written on loose paper **ON ONE SIDE ONLY**. Students should ensure their **name** and **school** is clearly written on the **first** answer sheet and that **all** pages are numbered. **EACH QUESTION ANSWERED must be started on a new page.** A standard formula booklet with standard physical constants may be used if desired.

Instructions: This paper will be invigilated over Zoom with all candidates starting at 9am, however please **do not discuss** any aspect of the paper on the internet until 9am Saturday 6th March.

Clarity: Solutions must be written legibly, in black pen (the papers are scanned), and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam.

Eligibility: The International Olympiad will be held during November 2021; all sixth form students are eligible to participate.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Training Dates and the IOAA (Bogota, Colombia, November 2021)

The team will be selected from Y12 students taking this paper **as well as** *Y12 students taking the Senior Physics Challenge paper in March. The best students that are eligible to represent the UK at the IOAA will be invited to attend the* **Training Camp** *to be held online from* **(Tuesday 6th April to Saturday 10th April 2021)**. *Astronomy material will be covered; problem solving skills and observational skills (telescope and naked eye observations) will be developed. At the Training Camp a Data Analysis exam along with a Round 3 theory paper will be sat. A team of five students (plus one reserve) will be selected for further training, mostly done remotely with (hopefully) additional training camps in the summer.*

Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Sun	$R_\odot$	$6.96 \times 10^{8}\,\mathrm{m}$
Radius of the Earth	$R_\oplus$	$6.37 \times 10^{6}\,\mathrm{m}$
Mass of the Sun	$M_\odot$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$M_\oplus$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_\odot$	$3.85 \times 10^{26}\,\mathrm{W}$
Stephan-Boltzmann constant	σ	5.67×10^{-8} J m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k_B	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck's constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Proton rest mass	m_p	1.67×10^{-27} kg
Electron rest mass	m_e	9.11×10^{-31} kg
Wien's displacement law	$\lambda_\mathrm{max}T$	2.90×10^{-3} m K
Avagadro's constant	N_A	6.02×10^{23} mol^{-1}

Basic calculus formulae:

Chain rule $$\frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = f'(g(x))g'(x)$$

Product rule $$\frac{\mathrm{d}}{\mathrm{d}x}(uv) = \frac{\mathrm{d}u}{\mathrm{d}x}v + u\frac{\mathrm{d}v}{\mathrm{d}x}$$

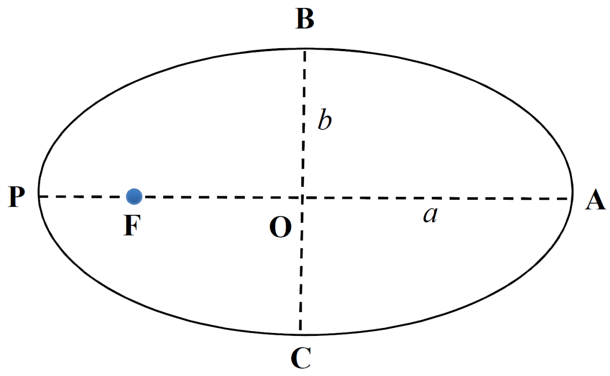
Quotient rule $$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right) = \frac{\frac{\mathrm{d}u}{\mathrm{d}x}v - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

Integration by parts $$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Standard integral $$\int \frac{1}{x}\,\mathrm{d}x = \ln|x| + C$$

Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

	$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
	$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
	$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
	$\mathbf{F}$	focus
	$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from **F**)
	$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from **F**)

Kepler's Third Law: For an elliptical orbit, the square of the period, T, of an object about the focus is proportional to the cube of the semi-major axis, a (as defined above), such that

$$T^2 = \frac{4\pi^2}{GM} a^3 \ ,$$

where M is the total mass of the system (typically dominated by the central object) and G is the universal gravitational constant.

Vis-Viva Equation: For an elliptical orbit, the speed v of an object at a distance r from the focus is related to the semi-major axis, a, total mass of the system, M, and universal gravitational constant, G, (as defined above), such that

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \ .$$

Magnitudes: The apparent magnitudes of two objects, m_1 and m_0, are related to their apparent brightnesses, b_1 and b_0, via the formula

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)} \ .$$

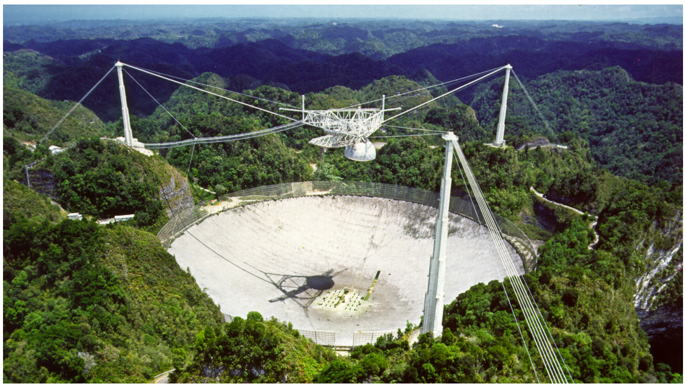
The absolute magnitude of an object, $\mathcal{M}$, is the same as its apparent magnitude when viewed from 10 pc, hence the relationship between apparent and absolute magnitude and distance is

$$m - \mathcal{M} = 5\log\left(\frac{d}{10}\right) ,$$

where d is measured in parsecs.

Qu 1. Mercury’s Rotation Period with Aricebo

In November 2020 the Aricebo Telescope at the National Astronomy and Ionosphere Centre (NAIC) in Puerto Rico was decommissioned due to safety concerns after extensive storm damage. First opened in November 1963, this brought an end to an illustrious contribution to radio astronomy where, with a dish diameter of 304.8 m (1000 ft), it was the largest radio telescope in the world until 2016. Its important discoveries range from detection of the first extrasolar planets around a pulsar to fast radio bursts, as well as a pivotal role in the search for extraterrestrial intelligence (SETI), however in this question we will explore its earliest major revelation: that Mercury was not tidally locked.



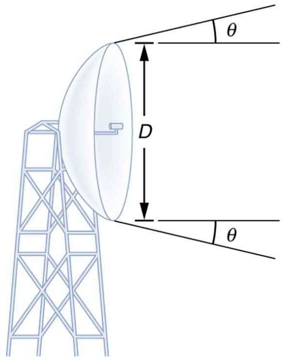


Figure 1: *Left*: The Aricebo telescope before it was damaged. Credit: NAIC.
Right: When transmitting a pulse from a radio telescope, diffraction prevents the beam from staying perfectly parallel and so the width of the beam increases by 2θ. Credit: OpenStax, College Physics.

Mercury had already been studied with optical and infrared telescopes, however the advantage of a radio telescope was that you could send pulses and receive their reflections. This radar-ranging technique had already been used with Venus to measure the distance to it and hence provide the data necessary for a definitive measurement of an astronomical unit in metres.

The radar echo from Mercury is much harder to detect due to the extra distance travelled and its smaller cross-sectional area (its radius is 2440 km). In April 1965, Pettengill and Dyce sent a series of 500 μs pulses at 430 MHz with a transmitted power of 2.0 MW towards Mercury whilst it was at its closest point in its orbit to Earth. In ideal circumstances the beam would stay parallel, however diffraction widens the beam as shown on the right in Fig 1.

a. Calculate the power of each echo received by the Aricebo telescope and hence determine the total number of photons in each echo, given the echo was detected 579.3 s after being transmitted and Mercury’s surface only reflects 6.5% of the incident radio photons. Assume $\theta = 0.16^\circ$ and the reflected photons from Mercury are scattered randomly within only the hemisphere facing Earth.

The signal-to-noise ratio of this echo was high enough that Doppler broadening of the received signal was reliably detected, allowing a determination of the rotation rate of Mercury. In August 1965 the same scientists sent 100 μs pulses and sampled the echo on short timescales as it returned. The strongest echo (received first) came from the point of the planet closest to the Earth (called the sub-radar point), with later echos coming from other parts of the surface in an annulus of increasing radius (see Fig 2).

Photons from the approaching side would be blueshifted to a higher frequency, whilst those from the receding side would be redshifted to a lower frequency. Hence, by measuring the Doppler shift and the time delay, you can map the rotational velocity as a function of apparent longitude and so can calculate the apparent rotation rate (as well as the direction of rotation and co-ordinates of the pole).

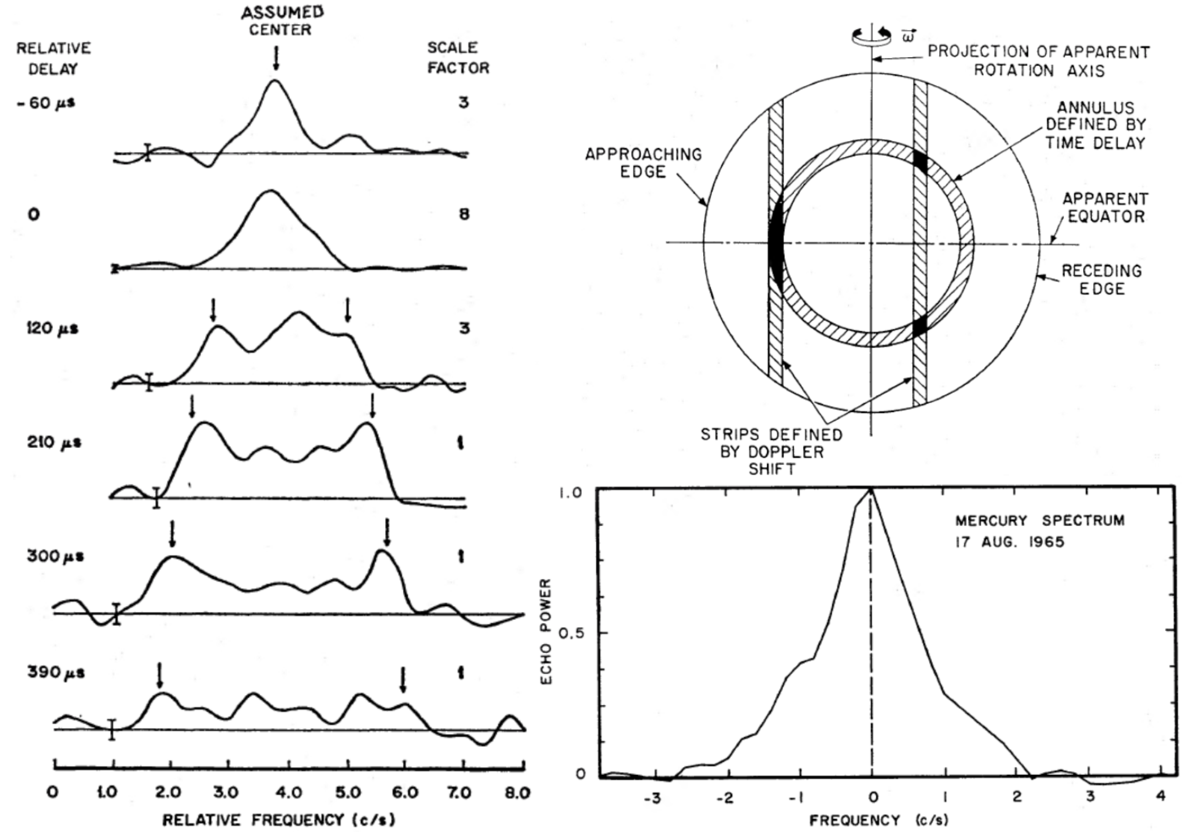


Figure 2: *Left*: Snapshots of the reflections of a single 100 μs pulse. The strength of the echo weakens as you move to later delays (as represented by the scale factor in the top right of each snapshot) and hence you need to use an annulus rather than detections from the horizon. The small arrows indicate the Doppler shifted frequency associated with intersection of the annulus with the apparent equator for each delay. The horizontal axis is in cycles per second (and so 1 c/s = 1 Hz). Credit: Dyce, Pettengill and Shapiro (1967).
Top right: The key principles of the delay-Doppler technique, looking at a cross-section of the planet. At the very centre is the sub-radar point (the point on the planet's surface closest to the Earth). As you move away from the sub-radar point the light has to travel further before it can be reflected, and hence the echo from those regions arrives later. The brightest point of any given annulus is where it intersects the apparent equator (due to the largest reflecting area), and so in each of the snapshots that is why the extreme Doppler shifts are boosted relative to the middle. Credit: Shapiro (1967).
Bottom right: The same as the snapshots, but this time summed over the first 500 μs of reflections. Here the difference between the extreme left and right frequencies reliably detected is ~ 5 Hz, but when corrected for relative motion of the Earth and Mercury it becomes the value given in part c. Credit: Pettengill, Dyce and Campbell (1967).

The Doppler shift with light is given as

$$\frac{\Delta f}{f} = \frac{v}{c} \,,$$

where Δf is the shift in frequency f, v is the line-of-sight velocity of the emitting object and c is the speed of light.

b. Averaging over a series of pulses from August 1965, after correcting for the relative motion of the Earth and Mercury and the rotation rate of the Earth during the observations, the difference between the frequencies of photons from the extreme left and right parts of an annulus received 500 μs after the initial echo was $\Delta f_{\text{total}} = 4.27$ Hz.

  (i) Given that the pulse was Doppler shifted twice (once when reflected, once again when received back at Aricebo) show that the rotational period of Mercury is ~ 60 days. Assume that the axis of rotation is normal to the plane of observations. [1 day = 24 hours]

  (ii) Mercury has a semi-major axis of 0.387 au. Rounding slightly if necessary, express the ratio orbital period : rotational period in a simple integer form (the integers should be < 10).

Ever since the first maps of Mercury's surface by Schiaparelli in the late 1880s, many in the scientific community believed that Mercury would be tidally locked and so always present the same hemisphere to the Sun. The reason they expected the rotational period to be the same as its orbital period (i.e. a $1 : 1$ ratio), rather like the Moon, is because it is so close to the Sun and the tidal torques causing this synchronicity are proportional to r^{-6} where r is the distance from the massive body. Given it is the closest planet to the Sun, it receives by far the largest torques, so the discovery it was in a different ratio was a complete surprise to many of the scientists at the time.

c. The eccentricity of the planet's orbit became the prime suspect as to why its actual ratio would be stable over long time periods.

   (i) Calculate how many times larger the tidal torque is when Mercury is at perihelion than when it is at aphelion, given the eccentricity of the orbit is 0.206.

   (ii) Assuming the tidal torque at perihelion is the dominating factor in setting Mercury's rotation rate, predict the rotational period of Mercury if it were to behave as though it was tidally locked when passing through perihelion. Compare this to the measured value and comment on validity of the assumption.

   (iii) Fig 3 shows the orientation of Mercury's axis of minimum moment of inertia (the axis the tidal torque acts upon) if the ratio had been $1 : 1$. Redraw this diagram but for the ratio found in part b (ii). Assume the initial orientation at perihelion looks the same in both cases (so only the other five positions are needed, separated equally in time) and that the planet both orbits and rotates in an anticlockwise direction. [Note: the orientation when it returns to perihelion may not be the same as it was initially.]

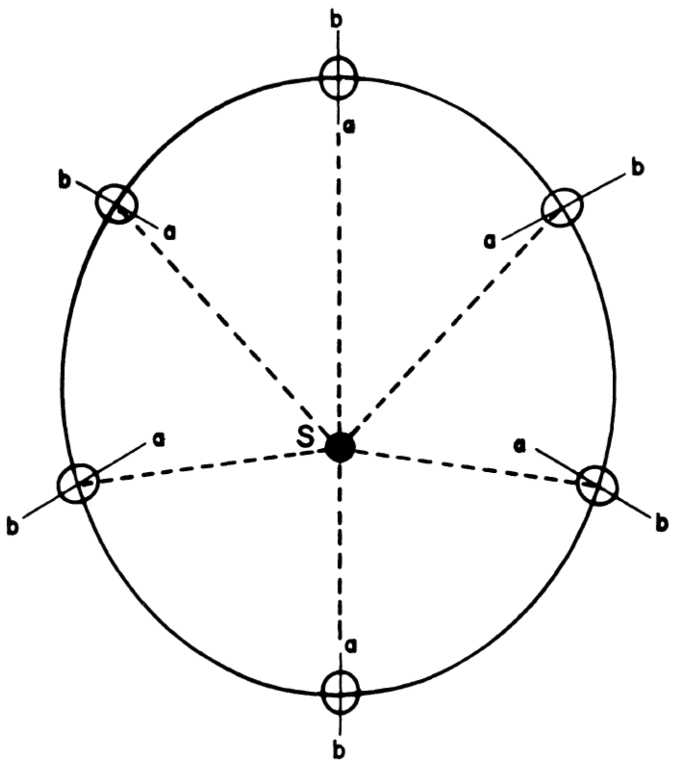


Figure 3: The orientation of Mercury's axis of minimum moment of inertia (the axis the tidal torque acts upon) displayed at six points in its orbit (equally spaced in time) if the ratio had been $1 : 1$. Credit: Colombo and Shapiro (1966).

Qu 2. Great Conjunctions and the Star of Bethlehem

On 21st December 2020, Jupiter and Saturn formed a true spectacle in the southwestern sky just after sunset in the UK, separated by only $0.102°$. This is close enough that when viewed through a telescope both planets and their moons could be seen in the same field of view (see Fig 4).

When two planets occupy the same piece of sky it is known as a conjunction, and when it is Jupiter and Saturn it is known as a great conjunction (so named because they are the rarest of the naked-eye planet conjunctions). The reason it happens is because Jupiter's orbital velocity is higher than Saturn's and so as time goes on Jupiter catches up with and overtakes Saturn (at least as viewed from Earth), with the moment of overtaking corresponding to the conjunction. The process of the two getting closer and closer together has been seen in sky throughout the year (see Fig 5).

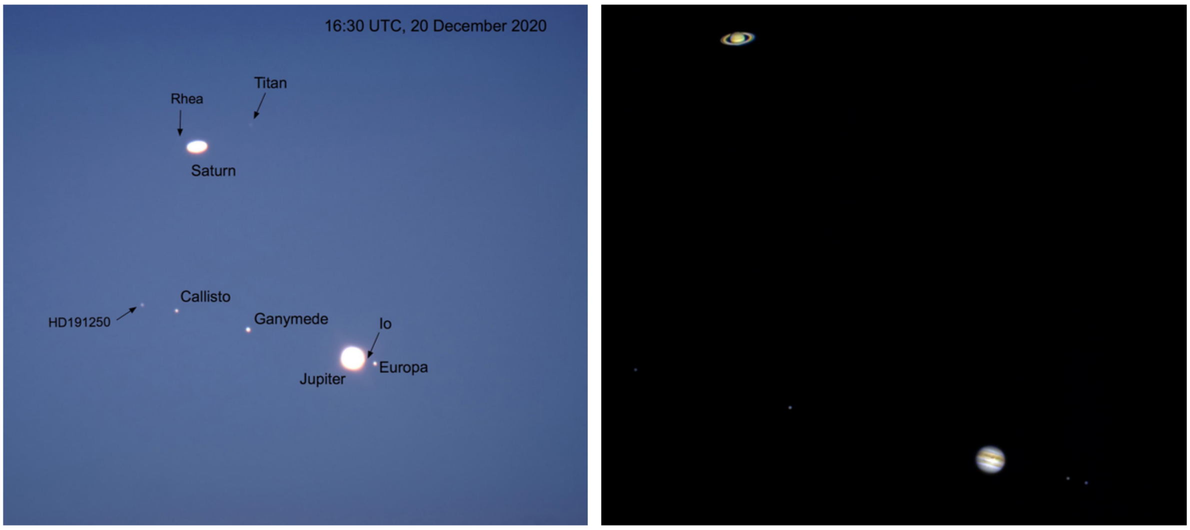


Figure 4: *Left*: The view of the planets the day before their closest approach, as captured by the 16" telescope at the Institute of Astronomy, Cambridge. Credit: Robin Catchpole.
Right: The view from Arizona on the day of the closest approach as viewed with the Lowell Discovery Telescope. Credit: Levine / Elbert / Bosh / Lowell Observatory.

a. In ideal observing conditions the two planets are far enough apart that they should be (just about) distinguishable to the naked eye, however to some observers in imperfect conditions they would appear as a single bright dot, brighter than either planet on its own.

(i) During the conjunction, the apparent magnitudes of Jupiter and Saturn were $m_J = -1.97$ and $m_S = 0.63$, respectively (ignoring dimming by the atmosphere). What would be the apparent magnitude of the two planets if they appeared to an observer as a single point? [Hint: it is not simply $-1.97 - 0.63 = -2.60$.]

(ii) Although they appeared close in angle, there was a very considerable distance between the two planets. At conjunction, Jupiter was 5.926 au from Earth whilst Saturn was 10.827 au (see Fig 5). If they were actually next to each other in space such that they could be treated as a single object, how far from the Earth (in au) would they need to be to have the same apparent magnitude as calculated in the previous part? For simplicity, assume that both planets can be modelled as (very low luminosity) stars so that the change in brightness is only due to changing the distance from the Earth (i.e. ignore the complications from the changing distance from the Sun affecting the number of reflected photons and the changing geometry affecting the illuminated fraction of the planet's surface).

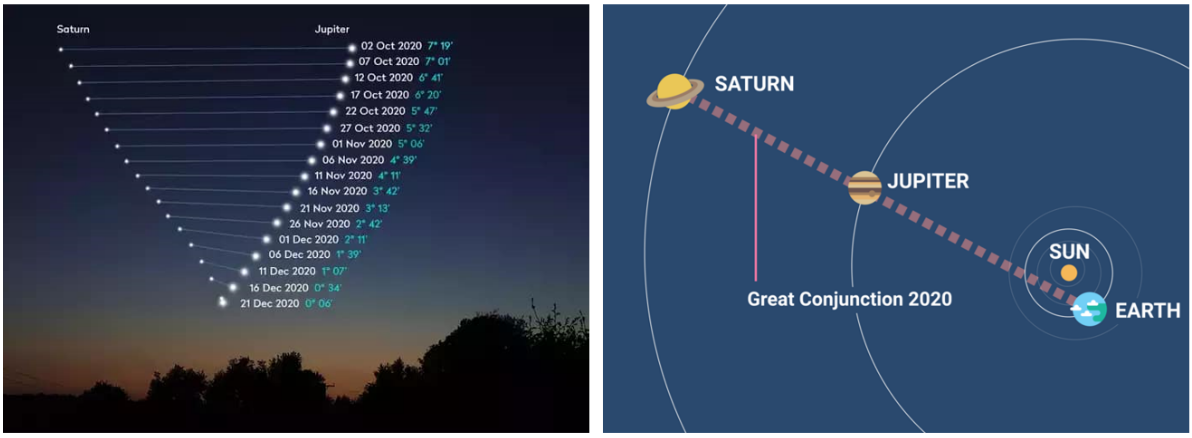


Figure 5: *Left*: Demonstrating how Jupiter and Saturn have been getting closer and closer together over the last few months. Separations are given in degrees and arcminutes ($1/60^{\text{th}}$ of a degree). Credit: Pete Lawrence. *Right*: The positions of the Earth, Jupiter and Saturn that were responsible for the 2020 great conjunction. The precise timing of the apparent alignment is clearly sensitive to where Earth is in its orbit. Credit: *timeanddate.com.*

The time between conjunctions is known as the synodic period. Although this period will change slightly from conjunction to conjunction due to different lines of perspective as viewed from Earth (see Fig 5), we can work out the *average* time between great conjunctions by considering both planets travelling on circular coplanar orbits and ignoring the position of the Earth.

b. Jupiter has a period of 4332.589 days and Saturn has a period of $10\,759.22$ days (where 1 day $= 24$ hours). Note: be careful as your calculations will be very sensitive to rounding errors.

  (i) Calculate the time between great conjunctions as viewed from the centre of the Solar System (this will be equal to the average synodic period). Give your answer in years (where 1 year $= 365.25$ days). [Hint: consider a reference frame rotating at the same rate as Jupiter.]

  (ii) Use your answer to predict the date (to the nearest day) of the next great conjunction.

  (iii) Some astronomers have suggested that the 'Star of Bethlehem' seen by the magi ('wise men') on their way to Jesus' birth was in fact a great conjunction. Use your average synodic period to find the date of the great conjunction in the first decade BC and give your answer to the nearest month. [Note: be careful with BC years as year 0 in your calculation is equivalent to 1 BC, since 31^{st} December 1 BC is followed by 1^{st} January 1 AD]

For circular coplanar orbits the centre of Jupiter's disc would pass in front of the centre of Saturn's disc every conjunction, and hence have an angular separation of $\theta = 0°$ (it is measured from the centre of each disc). In practice, the planets follow elliptical orbits that are in planes inclined at different angles to each other.

Fig 6 shows how this affects the real values for over 8000 years' worth of data, which along with different synodic periods between conjunctions makes it a difficult problem to solve precisely without a computer. However, after one synodic period Saturn has moved about $2/3$ of the way around its orbit, and so roughly every 3 synodic periods it is in a similar part of the sky. Consequently every third great conjunction follows a reasonably regular pattern which can be fit with a sinusoidal function.

c. By empirically fitting a sinusoidal function (which is assumed to be the same for each track, just with a fixed phase difference between them) and assuming all conjunctions are separated by the average synodic period, we can give rough estimations for the separations of any given great conjunction. Note: be careful as your calculations will be very sensitive to rounding errors.

(i) By reading off the graph, give an equation for Track A of the form $\theta = |D\sin(\frac{2\pi t}{\lambda} + \phi_A)|$, where t is the (decimalised) date in years, and D, λ, and $-\pi/2 < \phi_A \leq \pi/2$ are values that need to be determined. [Hint: ensure your function passes through the 2020 data point, and the function is decreasing as it does.]

(ii) Without having to read anything else off the graph, write down the equations for Tracks B and C, given the same restrictions on ϕ_B and ϕ_C.

(iii) State which track the 'Star of Bethlehem' great conjunction is on, and hence use the relevant equation to predict its separation. How does it compare to the 2020 great conjunction?

(iv) When is the next great conjunction at least as close as the 2020 one (i.e. $\theta \leq 0.102°$)? Give its year and the value of θ.

(v) Similarly, when was the last great conjunction at least as close as the 2020 one? Give its year and the value of θ.

(vi) Using your equations, calculate the probability that a great conjunction has $\theta \leq 0.102°$.

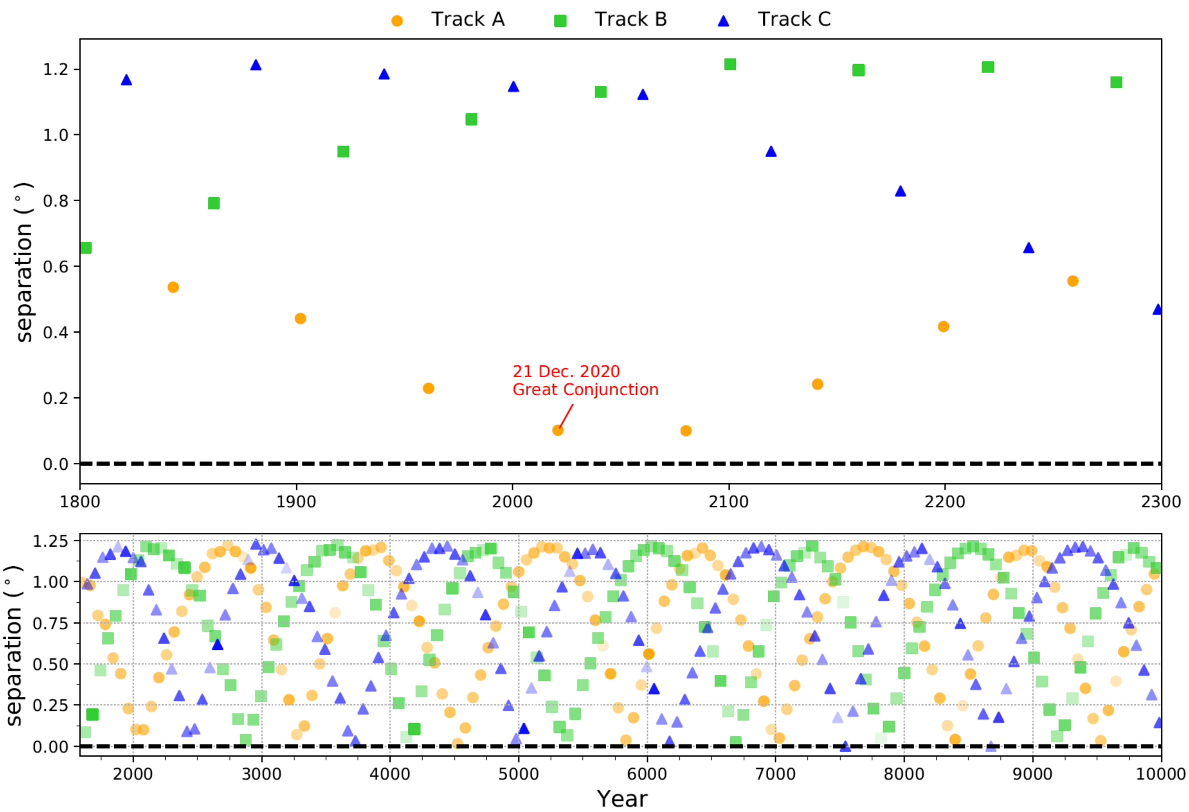


Figure 6: *Top*: All great conjunctions from 1800 to 2300, calculated for the real celestial mechanics of the Solar System. We can see that each great conjunction belongs to one of three different series or tracks, with Track A indicated with orange circles, Track B with green squares, and Track C with blue triangles.
Bottom: The same idea but extended over a much larger date range, up to 10 000 AD. It is clear the distinct series form broadly sinusoidal patterns which can be used with the average synodic period to give rough predictions for the separations of great conjunctions. The opacity of points is related to each conjunction's angular separation from the Sun (where low opacity means close to the Sun, so it is harder for any observers to see). Credit: Nick Koukoufilippas, but inspired by the work of Steffen Thorsen and Graham Jones / Sky & Telescope.

Qu 3. High Resolution Stellar Photos

The surface of the Sun has a temperature of $\sim$ 5700 K yet the solar corona (a very faint region of plasma normally only visible from Earth during a solar eclipse) is considerably hotter at around 10^6 K. The source of coronal heating is a mystery and so understanding how this might happen is one of several key science objectives of the Solar Orbiter spacecraft. It is equipped with an array of cameras and will take photos of the Sun from distances closer than ever before (other probes will go closer, but none of those have cameras).

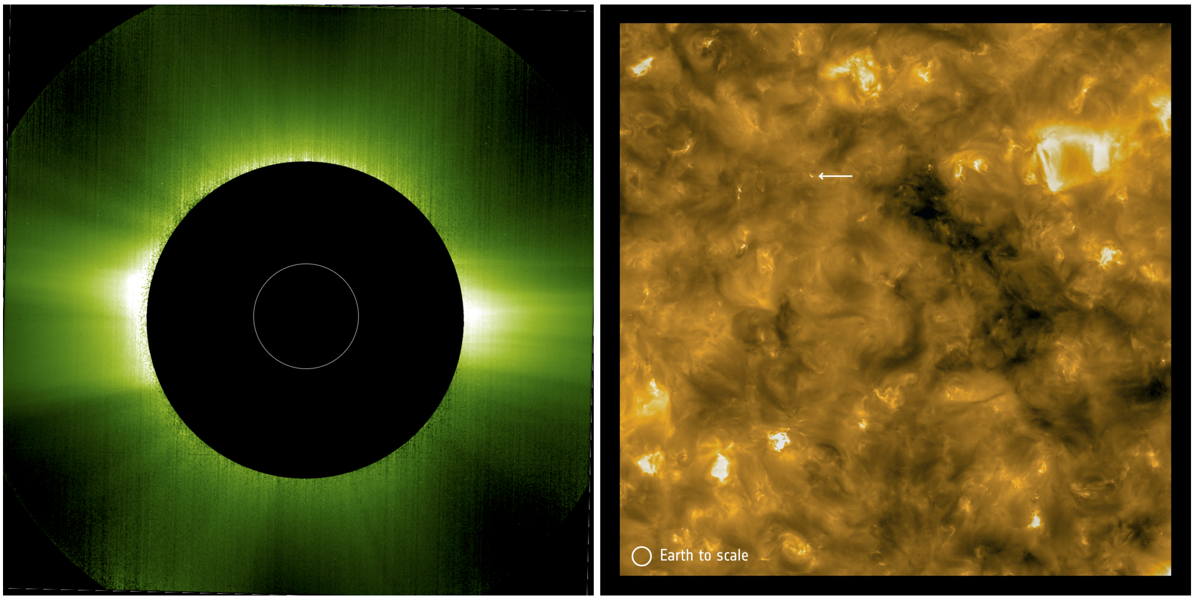


Figure 7: *Left*: The Sun's corona (coloured green) as viewed in visible light (580-640 nm) taken with the METIS coronagraph instrument onboard Solar Orbiter. The coronagraph is a disc that blocks out the light of the Sun (whose size and position is indicated with the white circle in the middle) so that the faint corona can be seen. This was taken just after first perihelion and is already at a resolution only matched by ground-based telescopes during a solar eclipse - once it gets into the main phase of the mission when it is even closer then its photos will be unrivalled. Credit: METIS Team / ESA & NASA
Right: A high-resolution image from the Extreme Ultraviolet Imager (EUI), taken with the HRI_{EUV} telescope just before first perihelion. The circle in the lower right corner indicates the size of Earth for scale. The arrow points to one of the ubiquitous features of the solar surface, called 'campfires', that were discovered by this spacecraft and may play an important role in heating the corona. Credit: EUI Team / ESA & NASA.

Launched in February 2020 (and taken to be at aphelion at launch), it arrived at its first perihelion on 15th June 2020 and has sent back some of the highest resolution images of the surface of the Sun (i.e. the base of the corona) we have ever seen. In them we have identified phenomena nicknamed as 'campfires' (see Fig 7) which are already being considered as a potential major contributor to the mechanism of coronal heating. Later on in its mission it will go in even closer, and so will take photos of the Sun in unprecedented detail.

a. When it reached first perihelion, radio signals from the probe took 446.58 s to reach Earth.

(i) Show that the spacecraft's perihelion is ~ 0.5 au, giving your answer to 4 s.f., and hence estimate the launch date, assuming the Earth's orbit is circular. Note that 2020 is a leap year and take 1 year $= 365.25$ days. [Hint: You may wish to use a numerical method.]

(ii) The circumference of an ellipse, unlike for a circle, has no closed form. A good approximation from Ramanujan (1914) is

$$C \approx \pi(a+b)\left[1+\frac{3j^2}{10+\sqrt{4-3j^2}}\right] \quad , \text{ where } \; j=\frac{a-b}{a+b} \, ,$$

and a and b represent the semi-major and semi-minor axes, respectively. Use this to work out the distance travelled by the probe between launch and perihelion to 4 s.f.

(iii) The exact solution for the circumference is

$$C = 4a\int_0^{\pi/2} \sqrt{1-e^2\sin^2 t}\,\mathrm{d}t \, ,$$

where e is the eccentricity and t is an angle in radians. The integral has no analytical solution, but can be written as an infinite series expansion and hence the expression becomes

$$C = 2a\pi[1 - De^2 - Ee^4 - \frac{5}{256}e^6 - \ldots] \, .$$

Find the values of D and E, given as fractions in their simplest terms, and hence calculate a new value for the distance travelled by the probe (also to 4 s.f.). Compare this to the approximation in the previous part and comment on your answer.
[A helpful infinite series expansion is $(1+x)^n = 1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+\ldots$*]*

The highest resolution photos are taken with the Extreme Ultraviolet Imager (EUI), which consists of three separate cameras. One of them, the Extreme Ultraviolet High Resolution Imager ($\mathrm{HRI_{EUV}}$), is designed to pick up an emission line from highly ionised atoms of iron in the corona. The iron being detected has lost 9 electrons (i.e. Fe^{9+}) though is called Fe X ('ten') by astronomers (as Fe I is the neutral atom). Its presence can be used to work out the temperature of the part of the corona being investigated by the instrument.

b. The energy density of black-body radiation, u, at temperature T is given as

$$u = \int_0^\infty \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/k_\mathrm{B}T)-1}\,\mathrm{d}\nu \, ,$$

and the number density of black-body radiation, n, at temperature T is given as

$$n = 16\pi\zeta(3)\left(\frac{k_\mathrm{B}T}{hc}\right)^3 \, .$$

Here h is Planck's constant, ν is the frequency, c is the speed of light, k_B is Boltzmann's constant, and ζ is the Riemann zeta function for which $\zeta(3) \approx 1.202$.

(i) The average energy per photon is given as $\bar{E} = \frac{u}{n} = \varepsilon k_\mathrm{B}T$. Find the numerical value of ε.
[Standard integral: $I = \int_0^\infty \frac{x^3}{\exp(x)-1}\,\mathrm{d}x = \frac{\pi^4}{15}$ *]*

(ii) Assuming the plasma of Fe ions is in thermal equilibrium with the photons, and that the average energy of the photons is equal to the ionisation energy of Fe X (which is 22 540 kJ $\mathrm{mol^{-1}}$), calculate the temperature of the plasma. Give your answer to 4 s.f.

The photons detected by HRI_{EUV} are emitted by a rearrangement of the electrons in the Fe X ion, corresponding to a photon energy of 71.0372 eV (where 1 eV $=$ 1.60×10^{-19} J). The HRI_{EUV} telescope has a $1000''$ by $1000''$ field of view (FOV, where $1^\circ = 3600'' = 3600$ arcseconds), an entrance pupil diameter of 47.4 mm, a couple of mirrors that give an effective focal length of 4187 mm, and the image is captured by a CCD with 2048 by 2048 pixels, each of which is 10 by 10 µm.

Although we are viewing the emissions of Fe X ions, the vast majority of the plasma in the corona is hydrogen and helium, and the bulk motions of this determine the timescales over which visible phenomena change. In particular, the speed of sound is very important if we do not want motion blur to affect our high resolution images, as this sets the limit on exposure times.

c. The Rayleigh criterion is

$$\theta = 1.22\frac{\lambda}{D} ,$$

where θ is the minimum angular diameter (in radians) that can be resolved by a telescope, λ is the observing wavelength and D is the diameter of the aperture of the telescope.
The speed of sound in a plasma is

$$v_s = \sqrt{\frac{\gamma k_B T}{\mu}}, \quad \text{for which} \quad \mu = \frac{m_p}{2X + 3Y/4 + Z/2} ,$$

where μ is the mean particle mass (X, Y, Z are the mass fractions of hydrogen, helium and elements heavier than helium, respectively), m_p is the mass of a proton, and γ is the adiabatic ratio.

(i) Determine the theoretical minimum angular diameter of an element resolvable by this optical system. Give your answer in arcseconds ($''$).

(ii) In practice, this is not achieved as the pixels are not small enough. Given that each picture element is spread across two pixels (in 1D) to allow adequate sampling, what is the actual minimum angle resolved on the CCD? Give your answer in arcseconds ($''$). [Hint: consider the geometry of the optical system and note that the angles are small enough that the small angle approximation can be used.]

(iii) Later on in its mission, Solar Orbiter will have a perihelion of 0.284 au. Calculate the physical size on the Sun (in km) of each picture element in an image taken with HRI_{EUV} as well as the FOV in units of $R_\odot$.

(iv) If the mass fractions of the surface of the Sun are $X = 0.7381$, $Y = 0.2485$, and $Z = 0.0134$, and treating the plasma as an ideal monatomic gas so that $\gamma = 5/3$, determine the speed of sound of the plasma (in km s^{-1}) at the base of the corona. Hence, by comparison to your answer from the previous part, estimate the upper limit on the length of an exposure to avoid motion blur in the plasma. You should ignore any motion blur from the relative motion of the spacecraft or the rotation of the Sun.

END OF PAPER

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