British Astronomy and Astrophysics Olympiad 2015-2016

Astronomy & Astrophysics Competition Paper

Monday 18th January 2016

Instructions

Time: 3 hours (approximately 35 minutes per question).

Questions: All five questions should be attempted.

Marks: The questions carry similar marks.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Students should ensure their name and school is clearly written on **all** answer sheets and pages are numbered. A standard formula booklet may be supplied.

Instructions: To accommodate students sitting the paper at different times, please **do not discuss** any aspect of the paper on the internet until 8 am Saturday 23rd January.

Clarity: Solutions must be written legibly, in black pen (the papers are photocopied), and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam paper.

Eligibility: The International Olympiad will be held during December 2016; all A Level students are eligible to participate, even if they will be attending university in December.

Training Dates and the International Astronomy and Astrophysics Olympiad

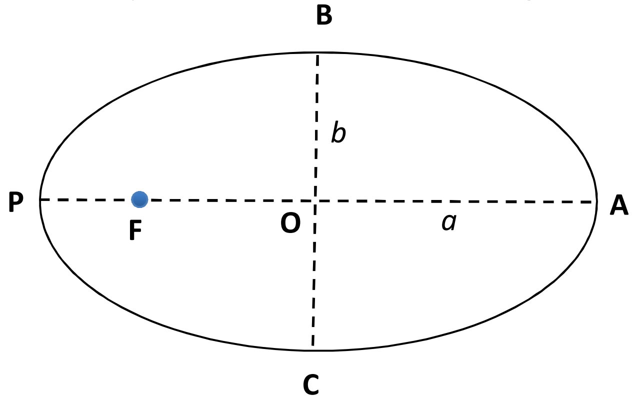
Following this round the best students eligible to represent the UK at the International Olympiad in Astronomy and Astrophysics (IOAA) will be invited to attend the ***Training Camp*** *to be held in the Physics Department at the University of Oxford, (****Monday 4th April –Thursday 7th April 2016****). Astronomy material will be covered; problem solving skills and observational skills (telescope and naked eye observations) will be developed. At the Training Camp a practical exam and a short theory paper will be sat. Five will be selected for further training. From May there will be mentoring by email to cover some topics and problems, followed by a* ***training camp at the beginning of July*** *and a weekend training camp in autumn.*

The IOAA this year will be held in Bhubaneswar, India, from ***9th to 19th December 2016****.*

Important constants

Speed of light in free space	c	3.00×10^8 m s^{-1}
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
Parsec	pc	3.09×10^{16} m
Astronomical Unit	AU	1.49×10^{11} m
Radius of the Earth	R_E	6.37×10^6 m
Radius of the Earth's orbit	r_E	1 AU
Radius of the Sun	$R_\odot$	6.96×10^8 m
Mass of the Sun	$M_\odot$	1.99×10^{30} kg
Mass of the Earth	M_E	5.97×10^{24} kg
Luminosity of the Sun	$L_\odot$	3.85×10^{26} W
Gravitational constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

a = OA (=PO) semi-major axis

b = OB (=CO) semi-minor axis

$e = \sqrt{1 - \frac{b^2}{a^2}}$ eccentricity

F – focus

P – periapsis (point nearest to F)

A – apoapsis (point furthest from F)

Kepler's Third Law: For an elliptical orbit, the square of the period, T, of orbit of an object about the focus is proportional to the cube of the semi-major axis, a (the average of the minimum and maximum distances from the Sun). The constant of proportionality is $4\pi^2/GM$, where M is the mass of the central object.

Magnitudes: The apparent magnitudes of two objects, m_1 and m_0, are related to their apparent brightnesses, b_1 and b_0, via the formula:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

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Qu 1. Asteroid Belt

In science fiction films the asteroid belt is typically portrayed as a region of the Solar System where the spacecraft needs to dodge and weave its way through many large asteroids that are rather close together. However, if this image were true then very few probes would be able to pass through the belt into the outer Solar System.

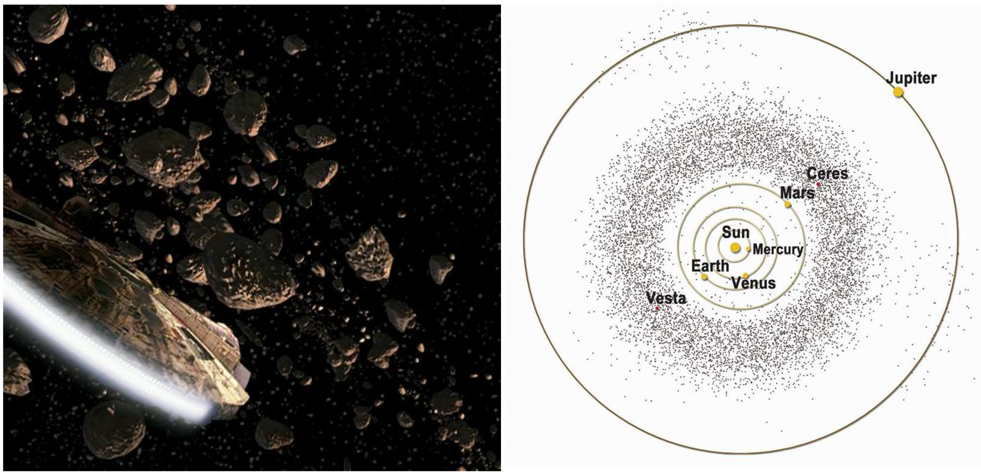

Figure 1 Artist conceptual illustration of the asteroid belt (left). Schematic of the Solar System with the asteroid belt between Mars and Jupiter (right).

This question will look at the real distances between asteroids.

a. Given that the total mass of the asteroid belt is approximately $M_{\text{belt}} = 1.8 \times 10^{-9}\, M_{\odot}$ calculate the radius of the object that could be formed, assuming it has a density typical of rock ($\rho = 3.0 \text{ g cm}^{-3}$). Compare this to the radius of the largest member of the asteroid belt, Ceres. (R_{Ceres} = 473 km)

b. The main part of the asteroid belt extends from 2.1 AU to 3.3 AU, and has an average angular width of 16.0°, as viewed from the Sun. Calculate the average thickness of the belt, and hence its total volume, V_{belt}.

c. Assuming this volume is uniformly filled by spherical rocky asteroids of average radius R_{av}, derive a relationship between the average distance between asteroids, d_{av}, and their radius R_{av}, remembering to keep the total mass equal to M_{belt}.

d. If R_{av} = 2.0 km, calculate d_{av}. How does this compare to the Earth-Moon distance? ($d_{\text{E}\to\text{M}} =$ 384,000 km)

An object with an apparent magnitude $m_0 = 0$ has an apparent brightness $b_0 = 2.52 \times 10^{-8} \text{ W m}^{-2}$.

e. Using the luminosity of the Sun, calculate the total power incident on an asteroid in the middle of the asteroid belt.

f. Assuming only 30% of that is reflected by its rocky surface, calculate the apparent magnitude of the asteroid when viewed from its nearest neighbour. Given that objects with $m > 6$ are too faint for the naked eye, would it be visible to an astronaut stood on the asteroid surface?

Qu 2. Supermoons

Figure 2 Supermoons at Perigee and Apogee. Image credit: John Gaughan/Pete Lardizabal/ WJLA.

A "supermoon" is a new or full moon that occurs with the Moon at or near its closest approach to Earth in a given orbit (perigee). The media commonly associates supermoons with extreme brightness and size, sometimes implying that the Moon itself will become larger and have an impact on human behaviour, but just how different is a supermoon compared to the 'normal' Moon we see each month?

Lunar Data:

Synodic Period	= 29.530589 days (time between same phases e.g. full moon to full moon)
Anomalistic Period	= 27.554550 days (time between perigees i.e. perigee to perigee)
Semi-major axis	= 3.844×10^5 km
Orbit eccentricity	= 0.0549
Radius of the Moon	= 1738.1 km
Mass of the Moon	= 7.342×10^{22} kg

In this question, we will only consider a full moon that is at perigee to be a supermoon.

a. Calculate how many days separate two supermoons.
b. Show that the difference in distance between the apogee and perigee is 4.22×10^4 km. (The data given in this question allows the mean orbital parameters to be calculated. Note that perturbations in the lunar orbit mean that the perigee and apogee continually change over the course of the year.)
c. Determine the difference in the angular diameter of a supermoon and a full moon observed at apogee. Thus, determine the percentage difference in the brightness of a supermoon and a full moon observed at apogee. (Ignore the effects of the Moon's orbital tilt with respect to the Earth.)
d. What change in magnitude does this brightness difference correspond to?
e. Suggest why it can be difficult to detect any differences in the brightness of supermoon compared to a 'normal' full moon when observing with the naked eye?
f. Calculate the gravitational field of the supermoon at the Earth. What fractional mass increase would a Moon at apogee need in order to create the same gravitational field?

Qu 3. Interstellar

In the science fiction movie *Interstellar*, the crops on Earth are failing, making farming difficult, and the existence of humanity is threatened. To save the human race, a crew of astronauts travelled through a wormhole in search of a new home and they sent encouraging data from planets near Gargantua, a supermassive black hole:

Miller's planet is the first planet in the system orbiting Gargantua. It is a water world with a similar composition to the Earth, covered in an endless shallow ocean. The planet's gravity is 130% of the Earth's, forcing human astronauts to move slowly and with some difficulty while on its surface. Being well within the tremendous gravitational field of Gargantua, time on the surface of Miller's planet passes very slowly relative to the rest of the universe: a single hour on Miller would equate to seven years back on Earth. Because of the planet's proximity to Gargantua, the immense gravitational pull from the black hole causes the planet to be afflicted by massive tidal waves as tall as 1 km. There is no sign of dry land on Miller, which may not exist due to the enormous erosive power of the planet's waves.

Figure 3 CGI model of a supermassive black hole and Miller's Planet. Credit: Interstellar.

The gravitational time dilation is given by:

$$t_0 = t_\mathrm{f}\sqrt{1 - \frac{2GM}{rc^2}}$$

where t_0 is the time for a slow-ticking observer within the gravitational field, t_f, is the time for a fast-ticking observer at an arbitrarily large distance from the massive object, M is the mass of the massive body, and r is the distance of the observer from the centre of the body.

You are asked to estimate the following:

a. The characteristics of the planet (mass and radius).
b. The mass of the supermassive black hole.
c. The orbital parameters of the planet (orbital radius and period).
d. The planet orbits the black hole, but the Hollywood director seems not to have checked his numbers carefully. In what way is this apparent from the values given and the results you have calculated? (the radius of the black hole is the event horizon, the value of r when $t_f \to \infty$.)

Qu 4. Dyson structures

Since its first light in 2009, the Kepler Telescope has been scanning the universe in search of habitable worlds beyond our Solar System. Kepler is designed to observe stars and look for tiny dips in their brightness. These dips, especially if they repeat, can be a sign the star has planets orbiting it. By measuring the timing and the size of the dips, scientists can learn a lot about the transiting planet.

During its routine observations of the star KIC 8462852, similar to our Sun (same radius and mass), the telescope observed something very unusual. A group of citizen scientists noticed that this star appeared to have two small dips in 2009, followed by a large dip lasting almost a week in 2011, and finally a series of multiple dips significantly dimming the star's light in 2013. The pattern of the dips indicates that a large, irregular-shaped object orbits the star. Some people have speculated that the star might be orbited by a giant alien megastructure, called a "Dyson structure". It is a structure that harnesses a star's energy to be used by a civilisation, like solar power, but on a massive scale. It would be composed of thousands of spacecraft that would be theoretically large enough to block out a significant portion of a star's light.

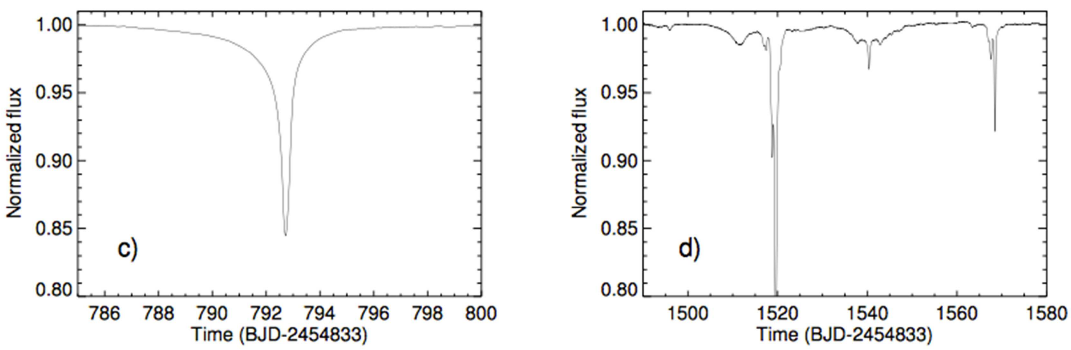

Figure 4 The light curves of KIC 8462852 showing two transits. The time is in days since a reference point. Credit: Boyajian et al, 2015, Planet Hunters X. KIC 8462852 - Where's the Flux?

a. Explain why the scientists believe that the object is unlikely to be a star or a planet.
b. Judging from the light curves in the plots above, what would be the area of the Dyson structure?
c. Based on the largest dips, what is the average distance of the Dyson structure from the star? Assume its mass is much smaller than the mass of the central star.

The problem of creating a Dyson structure is that it cannot be free floating in space. One possible solution is creating a cloud of solar sails. These objects would be in perfect balance between the gravity pulling them inwards, and the light pressure pushing them outwards. The luminosity (power output) of the star is $L_{\odot}$.

d. Assuming that the sails are made of a reflective material with reflectivity *R*, what is the pressure on the sail due to photon bombardment? (momentum of a photon is *E/c*)
e. What is the force exerted by the photons on the Dyson structure? Assume *R* = 1.
f. Assuming that the net acceleration of the solar sails is zero, what would be the mass of the structure?
g. Explain why the scenario of building such a structure is unrealistic.

Qu 5. Gravitational lensing

The deflection of light by a gravitational field was first predicted by Albert Einstein a century ago, suggesting that massive objects can bend light like a classical lens. This prediction was confirmed by Sir Arthur Edington in 1919, while observing a solar eclipse.

Consider a spherically symmetric object with mass M. This object will act like a lens, with an impact parameter b measured from the centre of the object. The angle of deflection due to the massive lens, given by General Relativity, is calculated as:

$$\alpha = \frac{4GM}{bc^2}$$

In a simplified model, the impact parameter may be seen as the shortest separation between the centre of the lens and the path of a particular light ray. The diagram below shows the geometric model of a gravitational lens (Figure 5). Light rays emitted from the source S being deflected by the lens are observed as images S_1 and S_2. The angles are *very* small.

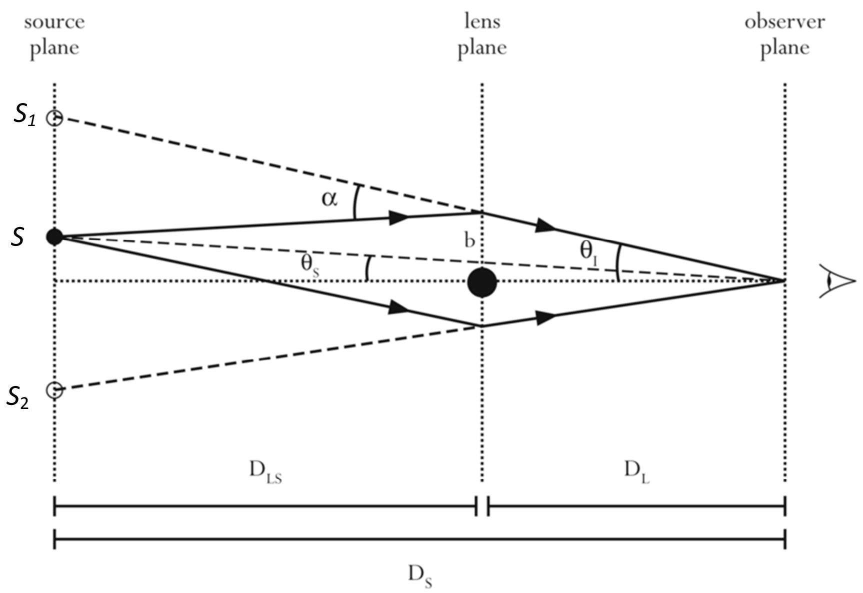

Figure 5 Schematic diagram of a gravitational lens.

a. Explain how Arthur Eddington might have used the gravitational lens effect to confirm the predictions of General Relativity.
b. Show that the source angle θ_S is related to θ_I, M, D_S, D_L via the expression:

$$\theta_S \approx \theta_I - \frac{D_{LS}}{D_L D_S}\frac{4GM}{c^2\theta_I}$$

c. For the special case in which the source is perfectly aligned with the lens such that $\theta_s = 0°$, a ring-like image (called an "Einstein ring") will occur. Find the angular radius, called the "Einstein radius" θ_E, of the ring.

d. Show that the time delay for a photon in the presence of an Einstein Ring is given by:

$$\Delta t = \frac{1}{2}\frac{D_S D_L}{D_{LS} c}\ \theta_E^2$$

$$[\text{You may use the approximation:} \frac{1}{\cos\alpha} = 1 + \frac{\alpha^2}{2}]$$

Extending the example of gravitational lensing into 3 dimensions, instead of two images of the source, sometimes multiple images of the source can be seen, arranged in a cross (called an "Einstein cross"). Last year, astronomers discovered a galaxy that is gravitationally lensed by a giant elliptical galaxy situated in a galaxy cluster in the foreground. Surprisingly, they discovered a supernova explosion (called the Refsdal Supernova) in the image of the lensed galaxy, arranged in an Einstein cross.

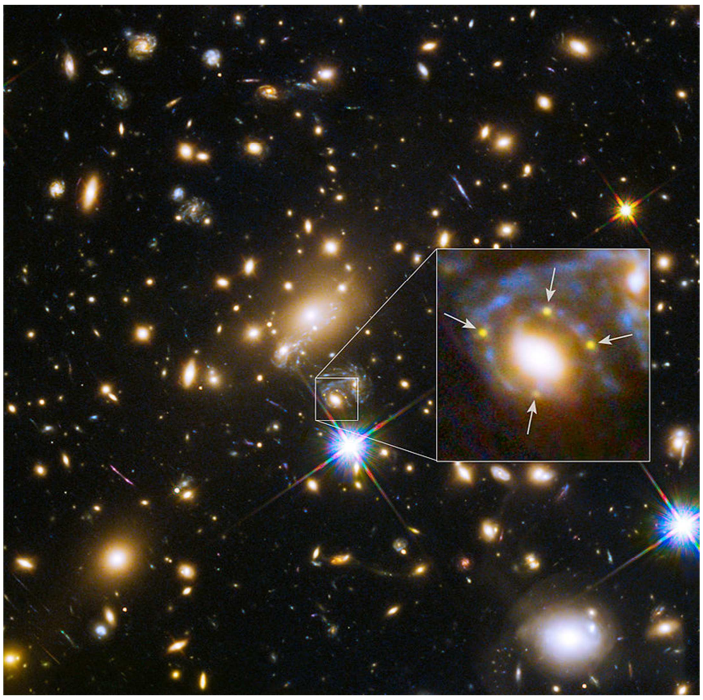

Figure 6 Multiply-lensed Refsdal supernova by a massive galaxy in the galaxy cluster MACS J1149.6+2223. Credit: Hubble/NASA/ESA/STSci/UCLA.

e. By knowing that the distance to the galaxy is 4.4 Gpc, and to the cluster is 2.0 Gpc, find the time delay caused by the lens for a photon from the supernova explosion. You can take the mass of the elliptical galaxy to be $M = 10^{12} M_{\odot}$. A Gpc is 10^9 parsecs.

f. Perhaps even more surprisingly, the astronomers realised that they were seeing the four images of the supernova at different time instances. In some of the pictures they took, images of the supernova were missing. Explain how this is possible.

End of Questions