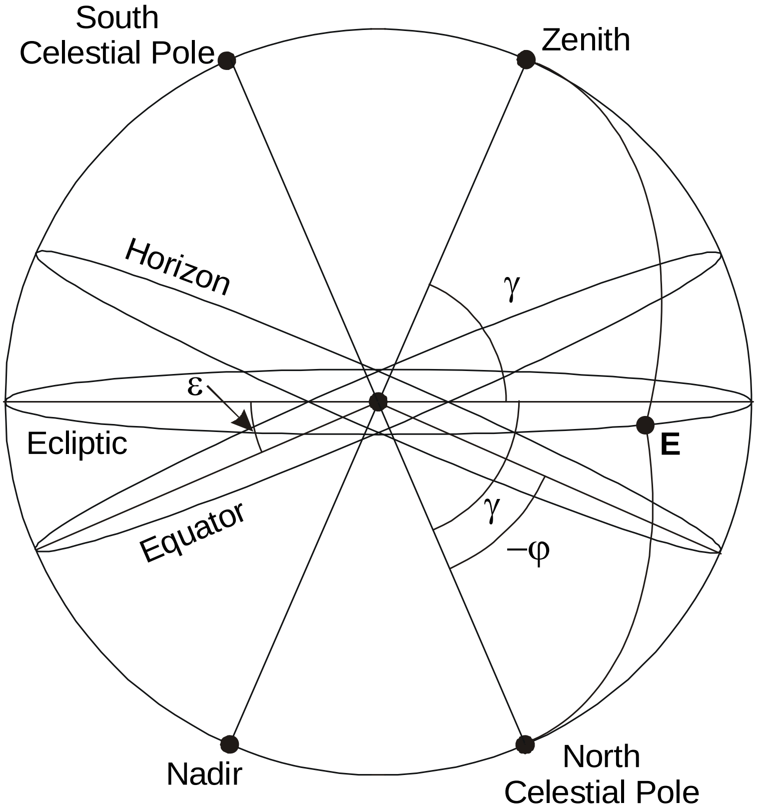
RUSSIAN OPEN SCHOOL
ASTRONOMICAL OLYMPIAD BY CORRESPONDENCE – 2008

PROBLEMS WITH SOLUTIONS

1. Problem. Observer is situated in the definite point on the Earth's surface. One definite moment he noticed that each point of ecliptic had met the mysterious property: the angular distance between this point and North Celestial Pole had been equal to the zenith distance of the same ecliptic point. Disregarding the refraction, please find the latitude of the observation point. (*O.S. Ugolnikov*)

1. Solution. The conditions described above are always met on the North Pole of the Earth (latitude +90°) where the North Celestial Pole coincides with the zenith and the angular distance from the North Celestial Pole is equal to the zenith distance not only for the points of the ecliptic, but for all points of the celestial sphere. But this is not the only solution.

If we observe somewhere far from the North Pole of the Earth, than the North Celestial Pole and the zenith are two different points of the celestial sphere. The problems conditions will be met if these two points are symmetric relatively the ecliptic line. It is seen in the figure for the example ecliptic point **E**.



Ecliptic is inclined to the equator by the angle ε equal to 23.4°. The angle between the northern polar direction (which is perpendicular to the equator) and the ecliptic plane is equal to

$$\gamma = 90° - \varepsilon = 66.6°.$$

Symmetry of North Celestial Pole and the zenith relatively the ecliptic means that the angle between the zenith direction and the ecliptic plane is the same. The ecliptic plane must be perpendicular to the plane containing the zenith and northern polar directions. Thus, the zenith distance of North Celestial Pole is equal to

$$z_P = 2\gamma = 133.2°.$$

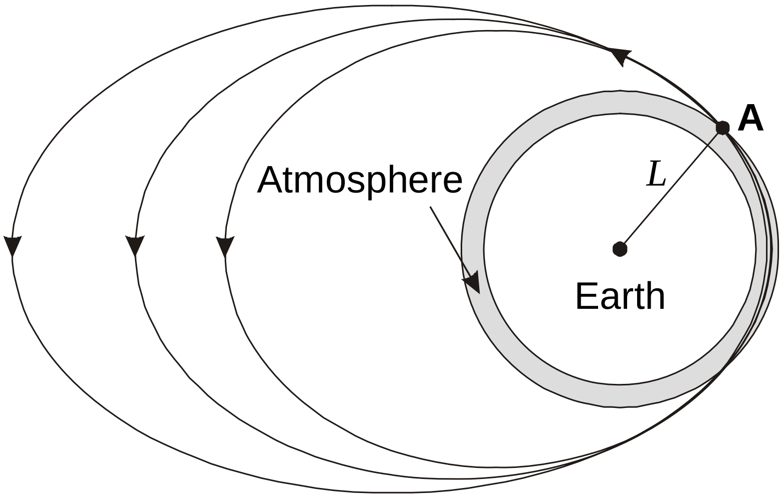
Northern Celestial Pole is below the horizon, and the observation point is in the Southern hemisphere of the Earth. The latitude is negative, its module is equal to the depth of Northern Celestial Pole:

$$\varphi = -(z_P - 90°) = 90° - 2\gamma = -90° + 2\varepsilon = -43.2°.$$

Finally, the problem condition can take place at the latitudes +90° (it always takes place there) and –43.2°.

2. Problem. The artificial satellite of the Earth has the mass equal to 100 kg and moves along the elliptical orbit with perigee altitude equal to 200 km and apogee altitude equal to 10000 km. Being close to perigee, the satellite is decelerated by the Earth's atmosphere. Please estimate the time, during which the satellite's orbit will become circular. The decelerating force of the atmosphere can be considered to be constant with the value 0.01 Newton, the path length of the satellite through the atmosphere each revolution is equal to the radius of the Earth. (*O.S. Ugolnikov*)

2. Solution. Let's explain why the orbit of the satellite will turn to circle. Being close to the apogee, the satellite is far from the Earth, it is not decelerated by the atmosphere and moves by the elliptical trajectory according to the Second Kepler law. Approaching the perigee, the velocity of the satellite increases and exceeds the circular velocity for this distance to the Earth. But here the satellite is being decelerated by the atmosphere. As we will see below, this deceleration is not too strong to lead the satellite to fall down or burn up in the atmosphere during the first revolution. But each revolution the satellite will loose the velocity and energy before the escape from the atmosphere.



Let L will be the distance from the center of the Earth to the upper border of the dense atmosphere layers, the atmosphere deceleration above this border can be neglected. The satellite velocity on this border is equal to

$$v^2 = GM\left(\frac{2}{L} - \frac{1}{a}\right),$$

where M is the Earth mass and a is orbit large semi-axis. Each revolution the values of v and a are decreasing. While the orbit is elliptical and satellite escapes from the atmosphere, its perigee altitude is changing slowly. Thus, the orbit of the satellite turns to the circle with the radius close to the perigee distance of the initial orbit. When the satellite does not escape the atmosphere, its trajectory will turn to spiral. The satellite will go down to the Earth increasing its velocity until it burn or fall down. But this stage of orbit evolution is not considered in this problem.

The following solution is the simplified method of the calculation of the time of orbit evolution, that is quite complicated problem in general. Let's consider two consecutive moments of the satellite's escape from the dense layers of atmosphere (the point **A** in the figure). Of course, the escape points are different, but their distances from the center of the Earth, *L*, are the same. Let's draw the relations between the velocities of the satellite at these moments and the values of major semi-axes of the orbit at (*i*) and (*i*+1) revolutions:

$$v_i^2 = GM\left(\frac{2}{L} - \frac{1}{a_i}\right),$$

$$v_{i+1}^2 = GM\left(\frac{2}{L} - \frac{1}{a_{i+1}}\right),$$

According to the constant energy law,

$$\frac{m}{2}(v_{i+1}^2 - v_i^2) = -F \cdot D,$$

Here *m* is the satellite mass, *F* is the deceleration force, *D* is the length of the satellite path through the atmosphere. According to the problem condition, the values of *F* and *D* are constant. Their multiplication is equal to 64 kJ, that is many times less than the kinetic energy of the satellite in perigee. Thus, the orbit will turn to circle slowly, during the large number of revolutions. The change values of major semi-axis Δa_i and orbital period ΔT_i are many times less than the values a_i and T_i themselves.

From the formulae above we obtain:

$$\frac{2FD}{GMm} = \frac{1}{a_{i+1}} - \frac{1}{a_i} \approx \frac{-\Delta a_i}{a_i^2}; \quad \Delta a_i = -\frac{2FDa_i^2}{GMm}.$$

Major semi-axis and orbital period are related with each other by the III Kepler law:

$$a_i^3 = T_i^2 \cdot \frac{GM}{4\pi^2}.$$

Mean change value of major semi-axis during (i)-revolution is the value of Δa_i divided by orbital period:

$$\psi_{i1} = \frac{\Delta a_i}{T_i} = -\frac{FD\sqrt{a_i}}{\pi m\sqrt{GM}}.$$

We can assume, that the change of major semi-axis in time will have a power law, we can find the number *n*, for which:

$$\psi_{in} = \frac{\Delta(a_i^n)}{T_i} = const_i.$$

Our aim is to find the number *n*. The parameter in the numerator of the last formula is equal to

$$\Delta(a_i^n) = a_{i+1}^n - a_i^n = (a_i + \Delta a_i)^n - a_i^n \approx n\Delta a_i a_i^{n-1}.$$

Here we had used the mathematical property of small value ρ:

$$(1+\rho)^n \approx 1+n\rho.$$

Finally, the change of value a_i^n per unit of time is equal to

$$\psi_{in} = \frac{\Delta(a_i^n)}{T_i} = \frac{\Delta a_i}{T_i} n a_i^{n-1} = -\frac{FDna^{(n-1/2)}}{\pi m\sqrt{GM}}.$$

We see that if $n=1/2$ than this value will not depend on time. Since the orbit evolution is much longer than the orbital period, we can assume the major semi-axis decrease as continuous process. The value of square root of major semi-axis will decrease by the linear law:

$$\sqrt{a} = \sqrt{a_0} - \frac{FD}{2\pi m\sqrt{GM}} t.$$

Here a_0 is the initial major semi-axis equal to

$$a_0 = R + \frac{h_P + h_A}{2} = 11470 \text{ km}.$$

Here R is the radius of the Earth, h_P and h_A are the altitude values in the perigee and apogee. During the orbit evolution the value of h_P is changing a little. The radius of circular orbit will be equal to

$$a_C = R + h_P = 6570 \text{ km}.$$

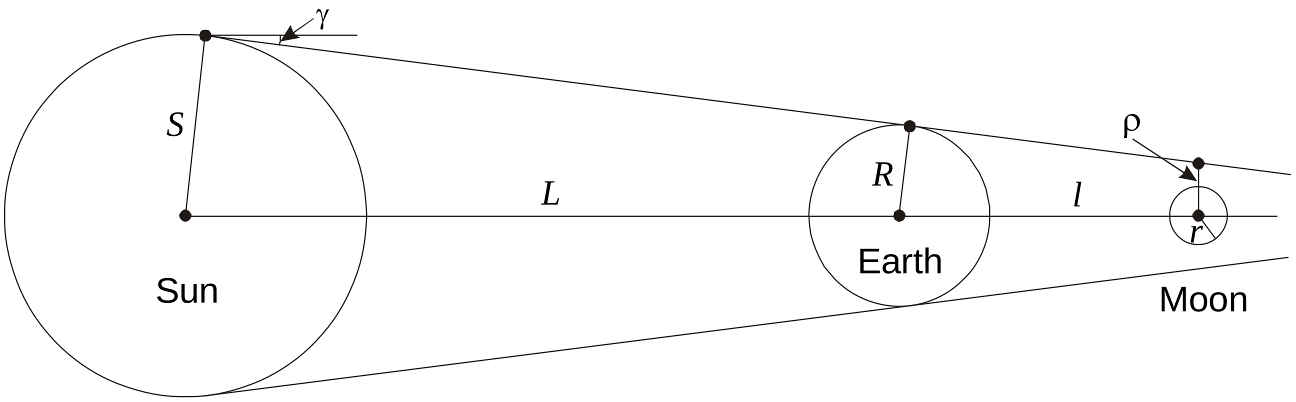
With account of $D=R$, the time of orbit evolution is equal to

$$T_C = \frac{2\pi m\sqrt{GM}}{FD}(\sqrt{a_0} - \sqrt{a_C}) = 1.6\cdot 10^8 \text{ sec}$$

or about 5 years.

3. Problem. The magnitude of total umbral lunar eclipse is equal to 1.865. Please find the duration of totality. The expansion of the umbra caused by atmosphere can be disregarded. (*O.S. Ugolnikov*)

3. Решение. The value of eclipse magnitude is too high. We have to define the distances between the Sun and the Earth (L) and between the Earth and the Moon (l) for which that value is possible. Let's assume that the eclipse is central and all three bodies are situated along one line. Their position is shown in the figure.

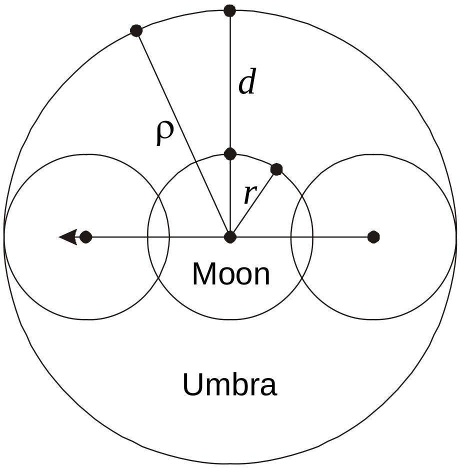


Let's define the umbra radius ρ depending on L and r. The edge of umbra cone tangent to Sun and Earth is inclined to the line "Sun-Earth" by the angle

$$\gamma = \frac{S-R}{L}.$$

This angle is close to the angular radius of the Sun, being quite small (it is equal to 0.26°) and we can use the trigonometric properties of small angle. In particular, we can state that its cosine is equal to unity. The umbra radius is equal to

$$\rho = R - \gamma l = \frac{R(L+l) - Sl}{L}.$$



As we can see in the figure, the central eclipse magnitude is equal to

$$F = 1 + \frac{d}{2r} = 1 + \frac{\rho - r}{2r} = \frac{\rho + r}{2r}.$$

Here r is the radius of the Moon. It is clear that the more the distance between the Sun and the Earth L, and the less the distance between the Sun and the Moon, l, the more the eclipse magnitude. We can see it using two initial formulae of solution. Let's assume that the value of L is maximal (1.017 a.u.), so the Earth reaches the aphelion point of the orbit. If we also assume that the value of l is average (384,400 km) than the eclipse magnitude will not exceed 1.832, that will not met the condition of the problem.

Substituting the minimal value of l (356,400 km) we obtain the maximal magnitude of the lunar eclipse (without account of atmospheric expansion) equal to 1.868. It nearly coincides with the one given in this problem. So, during the eclipse considered here the Moon was near the perigee point of the orbit and crossed the umbra along its diameter. The radius of the umbra is turned out to be equal to 4757 km.

The lunar spatial velocity at the distance l is equal to

$$v = \sqrt{GM\left(\frac{2}{l} - \frac{1}{a}\right)},$$

where a is the major semi-axis of the lunar orbit. Substituting the numbers, we obtain 1.095 km/sec. The umbra also moves relatively the Earth, its velocity is equal to

$$u = v_0 \frac{l}{L},$$

where v_0 is the aphelion orbital velocity of the Earth (equal to 29.3 km/s). The value of u is equal to 0.069 km/s, the direction is the same with the velocity of the Moon v. During the total eclipse the Moon covers the distance

$$D = 2(\rho - r),$$

and the total eclipse duration is equal to

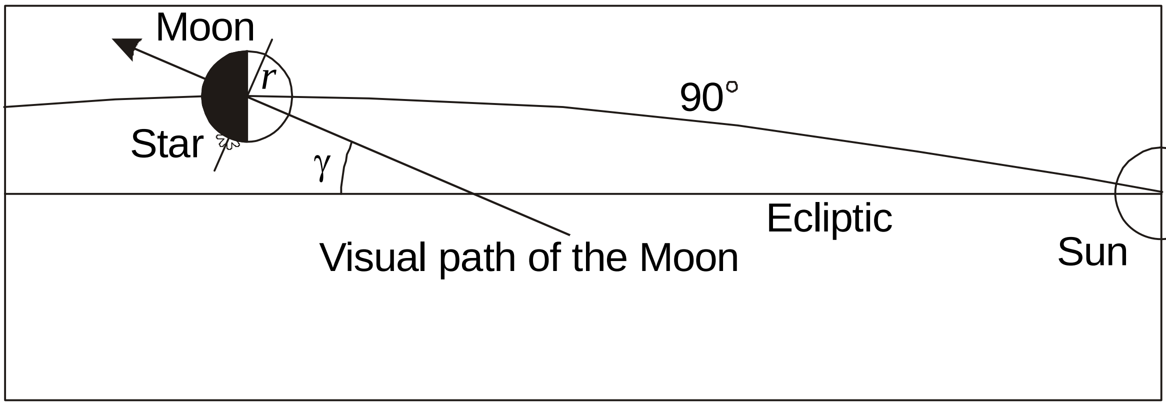
$$T = \frac{D}{v-u} = \frac{2(\rho - r)}{v-u},$$

or 5885 seconds or 1 hour and 38.1 minutes. Here we should notice that it is not the maximal duration of the total lunar eclipse that is reached during the apogee eclipses with smaller magnitude.

Note. In the ephemeredes of the lunar eclipses published in astronomical calendars and handbooks you can find the eclipses with magnitudes higher than 1.868. This fact is due to the account of atmospheric expansion of the umbra, which size is considered to be more than the geometrical value.

4. Problem. The grazing occultation of the star by the Moon is observed in the zenith at the Earth's equator. The Moon is exactly in the first quarter. Please find the maximum possible angular distance between the star being occulted and the closest "horn" of the Moon (the crossing point of limb and terminator) in the grazing moment. The orbit of the Moon can be considered to be circular. (*O.S. Ugolnikov*)

4. Solution. Let's draw the positions of Sun, Moon, star and ecliptic line during the occultation event. The cusps of the Moon are directed along the major circle of the celestial sphere connecting the Sun and the Moon. This circle is shown as the arc in the figure. We have to note that due to the inclination of the lunar orbit this circle does not coincide (in general) with ecliptic and visual path of the Moon on the celestial sphere.



Since the Sun is always situated on the ecliptic and the angular distance between the Sun and the Moon in the first quarter is equal to 90, cusps of the Moon are directed parallel to the ecliptic, not depending on the position of the Moon relatively the ecliptic. If the lunar visual motion relatively the star is directed parallel to the ecliptic, than the grazing occultation will be observed exactly at the "horn" point, the crossing of limb and terminator.

The angular distance between the star and the "horn" depends on the inclination of the lunar path to the ecliptic γ. So we have to define the maximal value of this angle. If we had observed from the center of the Earth, the problem would be much simpler. The value would be equal to the inclination of the lunar orbit, i, equal to 5.15°. It would be reached, when the Moon had crossed the node of its orbit. But really we observe from the surface of rotating Earth and the value of γ can exceed the value of i.

According to the problem, the occultation is observed in the zenith at the equator of the Earth. Since we have to find the maximal value of angle γ, we assume that the Moon is situated on the ecliptic crossing the orbit node. Let it be the ascending node.

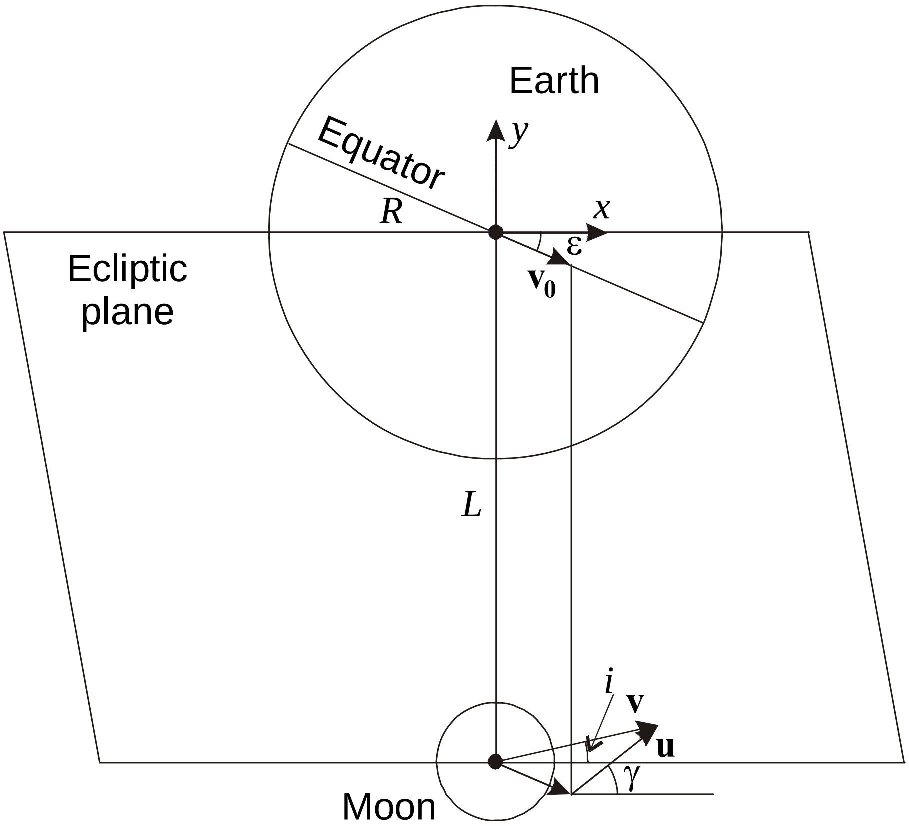


Figure shows the configuration of the Earth and the Moon on the ecliptic plane. The Moon crosses this plane by the angle i and moves northwards. We denote its velocity as v. Considering the orbit as circular, we find this velocity:

$$v = \sqrt{\frac{GM}{L}}.$$

Here M is the mass of the Earth, L – the distance between the Earth and the Moon. The velocity is equal to 1.02 km/sec. The observer is situated on the equator and moves with it by the angle ε (23.4°) to the ecliptic. The velocity is equal to

$$v_0 = \frac{2\pi R}{T}.$$

Here R is the radius of the Earth, T is the duration of sidereal day. The value is equal to 0.465 km/s. The velocity of the Moon relatively the observer is the vector difference of two velocities above:

$$\mathbf{u} = \mathbf{v} - \mathbf{v_0}.$$

The angle γ to find is the angle between vector **u** and the ecliptic plane. This angle is maximal if the vertical components of vectors **v** and $\mathbf{v_0}$ are opposite and the observer moves southwards. It is if the Moon is seen in the vernal equinox point. The situation is the same if the Moon is in descending angle and is seen in the autumn equinox point.

We define the coordinate system (x, y) as shown in the figure and make a projection of vector difference above on the axes of the system:

$$u_x = v\cos i - v_0 \cos\varepsilon,$$
$$u_y = v\sin i + v_0 \sin\varepsilon.$$

The angle γ is equal to

$$\gamma = \arctan\frac{u_y}{u_x} = \arctan\frac{v\sin i + v_0 \sin\varepsilon}{v\cos i - v_0 \cos\varepsilon} = 25.1^\circ.$$

We have to note that this angle is almost 5 times larger than for the case of geocentric observations. The angular distance between the star and "horn" is equal to

$$\sigma = 2\rho \sin\frac{\gamma}{2} = 2\frac{r}{L}\sin\frac{\gamma}{2}.$$

Here ρ is the angular radius of the Moon. Transforming this angle to the degree scale, we obtain 0.113° or 6.8′. If the grazing occultation occurs at the dark edge of the Moon (as shown in the first figure of solution), that angular distance will be enough to observe the event using the binocular or telescope even in the case of faint star.

5. Problem. The minor planet moves around the Sun in the ecliptic plane, never coming inside the orbit of the Earth. The conditions of its observations exactly repeat in 2 years, and its visible magnitude changes on 8^m with the same period. Please find the minimum possible value of the eccentricity of the asteroid's orbit. The asteroid is the smooth spherical uniform ball with constant surface albedo. Orbit of the Earth can be considered to be circular. (*O.S. Ugolnikov*)

5. Solution. In two years the Earth completes two revolutions around the Sun returning to the same point of the orbit. Since the conditions of asteroid observations are the same, it also returns to the same orbit point in two years. Since it does not come inside the orbit of the Earth, its orbit major semi-axis is not less than 1 a.u. and orbital period is not less than 1 year. The number of completed revolutions in 2 years is one or two.

But if this number was two and orbital period was equal to 1 year, asteroid would come inside the orbit of the Earth (in the case of elliptical orbit) or would be in the fixed position relatively the line "Sun-Earth" not changing the magnitude (in the case of circular orbit). Possible axial rotation of the asteroid does not change the picture, since the asteroid has the uniform surface. Finally, the orbital period of asteroid is equal to 2 years. According to Third Kepler law, the major semi-axis of the orbit is equal to $2^{2/3}$ or 1.587 a.u.

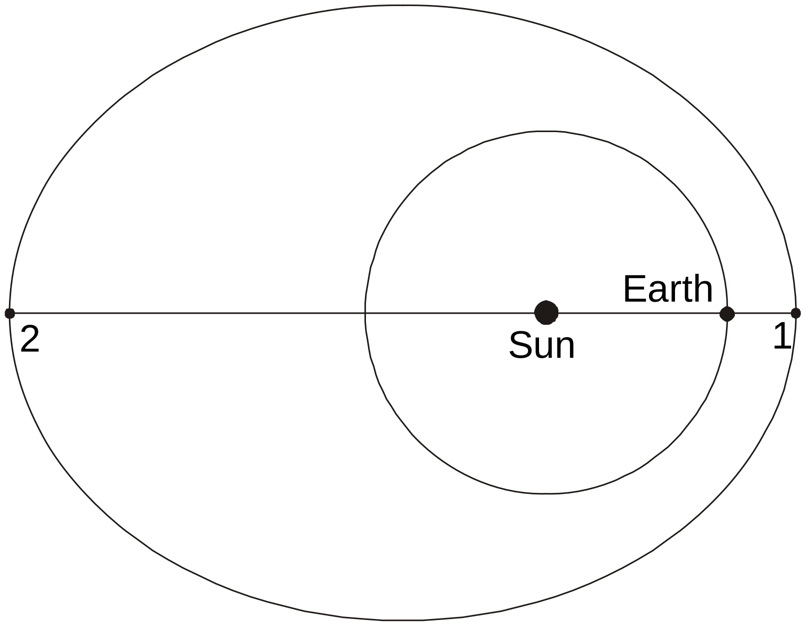
Asteroid is the uniform ball with smooth surface. This case its magnitude does not depend on the phase angle (the angle between the directions from the asteroid to the Sun and the Earth). The magnitude changes are related with the changes of distances between the Sun and asteroid and between the asteroid and the Earth. The asteroid brightness is reverse-proportional to the squares of both distances and the magnitude can be expressed as follows:

$$m = m_0 + 5\ \lg d + 5\ \lg r.$$

Here d and r are the distances from the asteroid to the Earth and to the Sun expressed in astronomical units, m_0 is the asteroid absolute magnitude (the magnitude for the case $d = r = 1$ a.u.).

Let e be the eccentricity of asteroid orbit. In the case of definite values of e and orbital period (two years) maximal amplitude of brightness changes will be reached in the case shown in the figure. During the opposition asteroid is in the orbit perihelion, position 1 in the figure. The values of d and r reach the minimum simultaneously:

$$d_1 = a\,(1 - e) - a_0,$$
$$r_1 = a\,(1 - e).$$



Here a_0 is the radius of Earth's orbit. The brightness of the asteroid reaches the maximal possible value. In one year the Earth returns to the same point, but asteroid is in aphelion and in the conjunction with the Sun. The values of *d* and *r* reach the maximum simultaneously:

$$d_2 = a\,(1 + e) + a_0,$$
$$r_2 = a\,(1 + e).$$

The brightness of asteroid reaches the absolute minimal value. Since the amplitude increases with the eccentricity, to find the minimal value of *e*, we have to find for the configuration described above for given amplitude – 8^m:

$$5 \lg d_2 + 5 \lg r_2 - 5 \lg d_1 - 5 \lg r_1 = 8,$$

$$\frac{d_2 r_2}{d_1 r_1} = \frac{(a(1+e)+a_0)\,a\,(1+e)}{(a(1-e)-a_0)\,a\,(1-e)} = K = 10^{1.6} = 39.8.$$

The solution of square equation gives the answer:

$$e = \frac{2(K+1)\,a - (K-1)\,a_0 \pm \sqrt{(K-1)^2 a_0^2 + 16Ka^2}}{2(K-1)\,a}.$$

Just one of two solutions (with the sign «–») has the physical sense ($0<e<1$). The eccentricity is equal to 0.284, the perihelion distance is equal to 1.137 a.u. We see that the asteroid really does not come inside the orbit of the Earth. If the eccentricity was less than 0.284, the amplitude (even in the case of opposition in perihelion and conjunction in aphelion) would be also less than 8^m. Thus, we had found the minimal possible value of eccentricity.

6. Problem. The observer on the Earth had measured the angular distance between the stars **X** and **Y**, both situated on the ecliptic, and obtained 30° with exactness 0.1″. The star **X** is situated westwards from the star **Y**, so it has less ecliptic longitude. During the observation moment both stars were situated westwards from the Sun, the ecliptic longitude difference of the Sun and star **X** was equal to 100°. Please find the angular distance between stars **X** and **Y** after three months. How will this distance change if we observe from the Sun? The parallax values of the stars **X** and **Y** are equal to 0.5″ and 0.2″, respectively. Please disregard the eccentricity of the Earth's orbit, self motions of the stars and all atmospheric effects. (*E.N. Fadeev*)

6. Solution. The coordinates of the stars change by two reasons: light aberration and parallax of the stars. We will analyze these effects separately.

Light aberration is caused by orbital motion of the Earth and the finite value of the speed of light. It moves the star towards the apex of the Earth's motion. The ecliptic longitude of the apex is 90° less than the one of the Sun. For the stars on the ecliptic the change of ecliptic longitude is equal to:

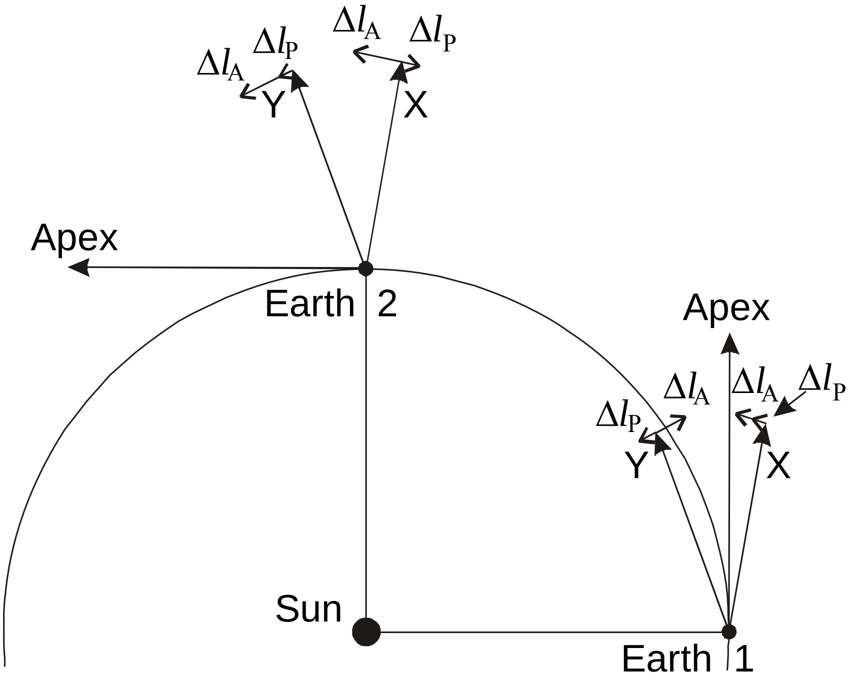
$$\Delta l_A = \frac{v}{c}\sin(a - l) = -k \cdot \cos(l_0 - l).$$

Here v is the velocity of the Earth, c is the light velocity, k is the aberration constant, equal to 20.5″ in degree scale, a is the apex ecliptic latitude, l_0 is the ecliptic latitude of the Sun, l is the ecliptic latitude of the star.

Parallax shift of the star is always directed towards the Sun. The change of ecliptic longitude is equal to

$$\Delta l_P = \pi \cdot \sin(l_0 - l).$$

Here π is the parallax of the star. Since the both shift values are too small, the common shift is the sum of these values, and we can us the observed values of star ecliptic longitude in these formulae.



According to the problem formulation, in the moment 1:

$$l_{01} - l_X = 100°,$$
$$l_{01} - l_Y = 70°.$$

The values of ecliptic longitude shifts owing to aberration and parallax are equal to:

$$\Delta l_{X1} = -k \cdot \cos(l_{01} - l_X) + \pi_X \cdot \sin(l_{01} - l_X) = +3.6'' + 0.5'' = +4.1''.$$
$$\Delta l_{Y1} = -k \cdot \cos(l_{01} - l_Y) + \pi_Y \cdot \sin(l_{01} - l_Y) = -7.0'' + 0.2'' = -6.8''.$$

The difference of ecliptic longitudes $(l_Y - l_X)_1$ in the moment 1 is equal to 30 and 10.9″ less than heliocentric difference $(l_Y - l_X)$. Thus, the heliocentric difference of longitudes is equal to 30°00′10.9″. This is the answer for the second question of the problem.

To answer on the first question, we see that in three months

$$l_{02} - l_X = 190°,$$
$$l_{02} - l_Y = 160°.$$

According to this,

$$\Delta l_{X2} = -k \cdot \cos(l_{02} - l_X) + \pi_X \cdot \sin(l_{02} - l_X) = +20.2'' - 0.1'' = +20.1''.$$
$$\Delta l_{Y2} = -k \cdot \cos(l_{02} - l_Y) + \pi_Y \cdot \sin(l_{02} - l_Y) = +19.3'' + 0.1'' = +19.4'.$$

The ecliptic longitude difference $(l_Y - l_X)_2$ is 0.7″ less than the heliocentric value $(l_Y - l_X)$ and equal to 30°00′10.2″.

7. Problem. In March 1997 we saw the bright comet Hale-Bopp with magnitude -1.5^m. Being observed from Earth, the brightest inner part of the comet's tail had the length about 10° and width about 1°. Imagine that the same time the spaceship with astronauts arrived to the comet and landed on its core at the side opposite relatively the Sun. Will the astronauts see the stars in the sky when they come to the surface of the core? (*O.S. Ugolnikov*)

7. Solution. To answer on the question, we have to look how do the brightness characteristics of expanded objects change when we fly to them or from them. If we approach to comet Hale-Bopp in 2 times for example, it will be brighter in 4 times and will have the angular square 4 times more than before. The surface brightness (or the magnitude of the angular square unit) will not change. If we come inside the expanded object, its emission will be seen from the major part of the sky, but the surface brightness will not be higher (actually it will be lower) than the one observed from large distance.

The surface brightness of the tail of comet Hale-Bopp is equal to -1.5^m per 10 square degrees. Each square degree contains 3600^2 square angular seconds. The magnitude of one square second is equal to

$$m = -1.5 + 2.5 \lg (10 \cdot 3600^2) = 18.8.$$

When we land on the comet surface at the side where the tail is visible, its surface brightness will be the same. But it is just $4\text{-}5^m$ brighter than the moonless night sky on the Earth. The comet night sky will be like evening sky during the nautical twilight. The human eye will see the stars up to 4m in these conditions. Finally, the astronauts will see the stars from the surface of the core of comet Hale-Bopp.

8. Problem. The star has the surface temperature 15000 K and the radius equal to 10 radii of the Sun. During the last 100 years this star produces the uniform stellar wind blowing with the velocity 20 km/s. This substance created the shell of gas and dust around the star with optical depth equal to 0.2. Please calculate the radii of inner and outer visible edges of the shell, find the dependence of the dust density in the shell on the distance from the star. Please find the temperature at the outer edge of the shell, mass of the shell and the mass loss rate of the star. The dust particles have the radius equal to 1 mkm, density equal to 3 g/cm^3 and fusion temperature equal to 1500 K. Consider that the mass of the gas is 200 times larger than the mass of the dust, but light absorption is caused only by dust. (*A.M. Tatarnikov*)

8. Solution. We know that the mass loss rate *M'* is constant. Let's divide the shell into the thin layers with the thickness Δr. Masses of each layer are the same, we denote is as ΔM. The number of particles is also the same for all layers. If the layer radius is R, than the density of the layer will be equal to

$$\rho\,(R) = \frac{\Delta M}{4\pi R^2 \Delta r} = \rho_{in}\left(\frac{R_{in}}{R}\right)^2.$$

Here R_{in} is the radius of the inner border of the shell and ρ_{in} is the density there. Last formula is the dependency of the density on the distance from the star. To find the temperature dependency, we denote dust particle radius as a. The energy falling to the particle is equal to the energy emitting by the particle to the surrounding space. If we denote the radius and temperature of the star as R_* и T_*, than we will have

$$\frac{4\pi\sigma T_*^4 R_*^2}{4\pi R^2}\cdot \pi a^2 = 4\pi\sigma a^2 T^4.$$

The temperature of particle is equal to

$$T = T_*\sqrt{\frac{R_*}{2R}}.$$

At the inner border of the shell the temperature must be equal to the dust fusion temperature. This case the dust will not reflect the emission closer to the star. Denoting the fusion temperature as T_0, we obtain

$$R_{in} = \frac{R_*}{2}\left(\frac{T_*}{T}\right)^2.$$

It is equal to 500 solar radii or 50 radii of the star. The outer border radius is more simple to calculate:

$$R_{out} = v\cdot t,$$

Here v is the stellar wind velocity (20 km/sec), t is the time of wind outflow (100 years). The radius is equal to $6.3\cdot 10^{10}$ km or 90000 solar radii or 9000 radii of the star. We see that outer border radius is many times more than the inner border radius. Physically it shows that the shell does create at the time t. The temperature of the outer border is equal to

$$T_{out} = T_*\sqrt{\frac{R_*}{2R_{out}}} = 110\ \text{K}.$$

To find the mass of the shell, we have to express the optical depth of the shell. We assume that the particles absorb the emission as the black balls with radius a. The probability of the photon to be absorbed in the layer with radius R and thickness ΔR, or the optical depth of the layer, is equal to

$$\Delta\tau\,(R) = n(R)\Delta R\cdot \pi a^2 = n_{in}\left(\frac{R_{in}}{R}\right)^2 \pi a^2 \Delta R.$$

Here $n(R)$ is the particle concentration at the distance R, n_{in} is their concentration at the inner border of the shell. Total optical depth is the sum of the optical depths of all layers. It is expressed as the integral

$$\tau = \int_{R_{in}}^{R_{out}} d\tau\,(R) = \int_{R_{in}}^{R_{out}} n_{in}\left(\frac{R_{in}}{R}\right)^2 \pi a^2 dR = \pi a^2 n_{in} R_{in}^2\left(\frac{1}{R_{in}} - \frac{1}{R_{out}}\right) \approx \pi a^2 n_{in} R_{in}.$$

Here we take into account that the outer radius of the shell is sufficiently larger than the inner radius. The value of optical depth is known, and we can use this formula to calculate the concentration n_{in}. The number of particles in thin layer with radius R and thickness ΔR is equal to

$$\Delta N(R) = n(R)\cdot 4\pi R^2 \Delta R = n_{in}\cdot 4\pi R_{in}^2 \Delta R = \frac{4R_{in}}{a^2}\Delta R.$$

This value does not depend on radius, since the mass outflow is constant. Taking it into account and knowing that outer radius is many times less than the inner one, we obtain

$$N = \frac{4\tau R_{in} R_{out}}{a^2}.$$

To find the total mass, we remember that the mass of gas is K times more than the mass of dust. If ρ_0 is the dust density, than the shell mass can be expressed as follows:

$$M = \frac{4}{3}\pi a^3 \rho_0 \cdot NK = \frac{16\pi\tau R_{in} R_{out} a \rho_0 K}{3}.$$

It is equal to $4\cdot 10^{25}$ kg or $2\cdot 10^{-5}$ of the solar mass. This mass was released in 100 years, so the mass loss rate is equal to $4\cdot 10^{23}$ kg or $2\cdot 10^{-7}$ solar masses per year.

9. Problem. The gamma-ray bursts sometimes happen in the distant galaxies. These are the short (about several seconds) bursts of gamma-ray emission with average energy of the photon equal to 1 MeV. To be registered on the Earth, the flux of such photons must be not less than 50 phot/(cm^2·s). The luminosity of the burst is equal to 10^{49} ergs per second, this energy is released inside two opposite cones with angle at the top equal to 10°. The gamma-ray bursts are registered on the Earth once a week. What is the frequency of gamma-ray bursts in one definite galaxy? How much times more or less bursts we would see, if the cones of their emission were two times narrower? (*M.E. Prokhorov*)

9. Solution. Let's find which part of the sphere is covered by two cones of gamma-ray emission. These cones draw two circles with radius 5° or 0.087 radians. We denote this angle as ρ. This angle is quite small and we consider the circles as plane figures. The part of the sphere covered by cones is equal to the ratio of circles and sphere squares:

$$b = \frac{2\pi\rho^2}{4\pi} = \frac{\rho^2}{2} = 0.004.$$

The energy of gamma-ray photon is equal to 1 MeV or $1.6\cdot 10^{-6}$ ergs. Thus, the gamma-ray source emits $6\cdot 10^{54}$ photons. We denote this value as J_0. We find the maximal distance to the source to be observed by the device with sensitivity E:

$$E = \frac{J_0}{4\pi b R^2}; \quad R = \frac{1}{2}\sqrt{\frac{J_0}{\pi b E}} = \frac{1}{\rho}\sqrt{\frac{J_0}{2\pi E}}.$$

Here we assume that the Earth is situated inside one of two emission cones. Thus, we can detect the gamma-ray burst at the distance up to 500 Mpc. The change of flux due to the Universe expansion is not sufficient at this distance.

The average concentration of galaxies in the Universe is equal to 0.01 Mpc^{-3}. We can obtain this value from the total number of galaxies in the Universe (10^{10}) and the size of the Universe (10 Gpc). The number of galaxies inside the sphere with radius R is equal to

$$N = \frac{4}{3}\pi R^3 n = \frac{n}{6\sqrt{\pi}}\left(\frac{J_0}{bE}\right)^{3/2} \sim 5\cdot 10^6.$$

We see the bursts with the frequency F equal to 1 burst per week or 50 bursts per year in this number of galaxies. But we cannot see all the bursts in these galaxies as we cannot always be inside the emission cones. Total frequency of burst in these galaxies is equal to

$$F_0 = \frac{F}{b}$$

or 12500 bursts per year. The bursts frequency in one galaxy is equal to

$$F_1 = \frac{F_0}{N} = F\,\frac{6\sqrt{\pi b}}{n}\left(\frac{E}{J_0}\right)^{3/2} = F\,\frac{3\sqrt{2\pi}}{n}\left(\frac{E}{J_0}\right)^{3/2}\rho.$$

or once in 400 years. We will see 1/250 of these bursts, so the frequency of observable bursts in one galaxy is once in 100,000 years.

We see that this value in the formula is proportional to the square root of b or proportional to the cone angular radius ρ. If the radius was two times less, the total frequency of the bursts in one galaxy would be two times less. But the value of F_0 is reverse proportional to the square root of b, so the total number of observable bursts would increase in two times!

It is easy to explain. If the cone radius was two times less, we could see two times more distant gamma-ray bursts. They would fulfill 8 times larger volume in the Universe. The probability to be inside the cones would be 4 times less. So, the frequency of gamma-ray burst observations would increase in two times.

10. Problem. We know that the temperature of Cosmic Microwave Background in the direction with Galactic coordinates l = 264° and b = 48° is maximal, being by ΔT = 3.35 mK more than average value. Please find the velocity of our Galaxy as a whole relatively the Cosmic Microwave Background. (*E.N. Fadeev*)

10. Solution. Firstly we have to find the velocity of Sun relatively the Cosmic Microwave Background (CMB). The temperature change is related with the Doppler effect:

$$\frac{\lambda_0 - \lambda}{\lambda} = \frac{v}{c}.$$

Here λ_0 is the average (through the sky) wavelength of the CMB maximum, λ is the one for the direction described above, v is the velocity of the Sun relatively CMB. Since the CMB emission is thermal, the wavelength of maximal emission is reverse proportional to the temperature:

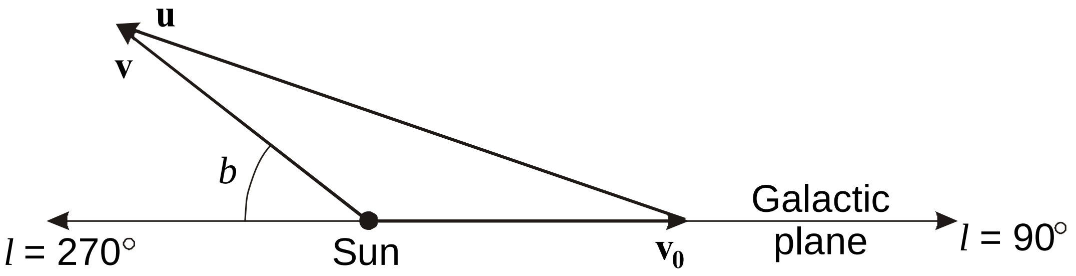
$$\lambda\,(\mathrm{cm}) = \frac{0.29}{T}$$

The velocity is equal to

$$v = c\cdot\frac{T - T_0}{T} = c\frac{\Delta T}{T}.$$

Thus, the Sun moves relatively CMB with the velocity 368 km/sec in the direction with galactic coordinates $l = 264°$ and $b = 48°$. But the Sun moves relatively the center of the Galaxy with the velocity v_0 equal to 220 km/sec and directed to the point with galactic coordinates l_0 = 90° and b_0 = 0°. The vector of common Galaxy velocity relatively CMB is equal to

$$\mathbf{u} = \mathbf{v} - \mathbf{v_0}.$$



We see that the galactic longitude *l* corresponding the vector **v** is close to 270°, and we can assume that all three vectors are in the figure plane perpendicular to the galactic plane. This case the value of the velocity *u* can be calculated as follows:

$$u = \sqrt{v^2 + v_0^2 - 2vv_0 \cos(180^\circ - b)}.$$

It is equal to 540 km/sec. If we take into account the difference of *l* and 270°, the answer will be the same with the exactness of 1 km/sec.