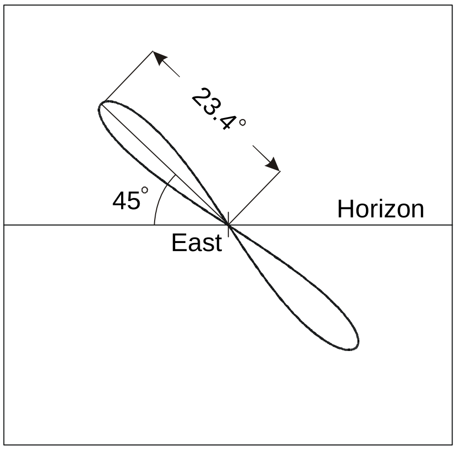
RUSSIAN OPEN SCHOOL ASTRONOMICAL OLYMPIAD BY CORRESPONDENCE – 2005

PROBLEMS WITH SOLUTIONS

1. Problem. The artificial satellite is rotating around the Earth along a circular orbit lying in the ecliptic plane. Being observed from city of Krasnodar (latitude +45°) this satellite and the vernal equinox point always rise simultaneously. Sometimes the satellite is visible above the southern horizon. What is its altitude above the horizon in this moment? What is the radius of satellite's orbit? Refraction and daily parallax of the satellite should be ignored.

1. Solution. Anywhere on Earth (except poles) the vernal equinox point rises once in a sidereal day, about 23 hours and 56 minutes after previous rise. The moments of rise of artificial satellite are in 23 hours 56 minutes one from another. Since we ignore the parallax shift of the satellite, it is always situated at the ecliptic. At the latitude of Krasnodar (+45°) ecliptic cannot coincide with horizon and being the major circle of celestial sphere crosses the horizon at two opposite points. One of them is rising vernal equinox point, another point is autumn equinox point. Thus, rising satellite is situated at one of these points. As the sidereal day gone, the satellite rises in the same point of the sky (being in the second point, it would not rise, but set under the horizon, since every moment just a half of the orbit is seen above the horizon).

It is obvious that the satellite rotating around the Earth cannot be immovable relatively stars. Let us consider the conditions when the satellite comes back to the same point in one sidereal day. It can take place if the rotational period is equal to one sidereal day and the rotation direction is the same with the rotation of the Earth. This orbit is like geo-stationary, but the satellite rotates not in equatorial, but in the ecliptic plane. Being observed from the Earth, the satellite draws the narrow 8-like figure with the size 47°. If it is situated at the vernal equinox point during its rise, it will rise every sidereal day at the east (see the figure).



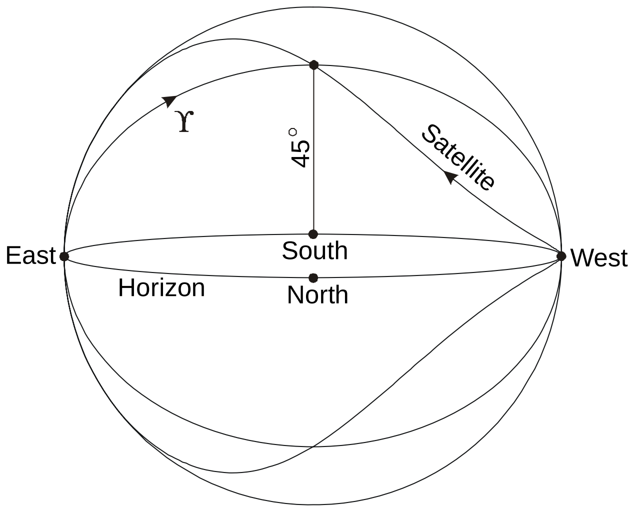
But this picture contradicts with the conditions of the problem, since the satellite is always situated in the eastern part of the sky and is never observed above the southern horizon. We have to consider one more situation, when satellite completes the revolution around the observer on the Earth in one sidereal day, rising once and setting once in this period. Let us designate the synodic period of the satellite as S, sidereal period as T, and the duration of sidereal day as T_0. These three values are connected by the relation

$$\pm\frac{1}{S} = \pm\frac{1}{T} - \frac{1}{T_0}.$$

The sign "+" before the values of $(1/S)$ and $(1/T)$ means the counterclockwise rotation direction (the same with the rotation of Earth), the sign "–" means the clockwise direction. Since the values of S and T_0 are the same, the sign "–" cannot be before the value (1/*S*), otherwise the value of *T* will turn to infinity. Since the value of *S* is positive, the solution will exist only for the following signs:

$$\frac{1}{S} = \frac{1}{T} - \frac{1}{T_0},$$

The sidereal period of the satellite *T* is turned out to be equal to the half of sidereal day or 11 hours and 58 minutes. The satellite rotates counterclockwise two times faster than Earth, rising at the west and setting at the east. During its rise the satellite will be situated in the *autumn* equinox, which will be setting in the same moment. Moving from the west to the east, the satellite will go towards the vernal equinox point risen at the east simultaneously with the satellite (see the second figure). In 5 hours and 59 minutes the satellite will complete a half of the revolution, come to the vernal equinox point, which will culminate at the altitude 45° above the southern horizon. We have found the answer to the first question of the problem.



To answer on the second question, we use the General Third Kepler law. The orbit radius is equal to

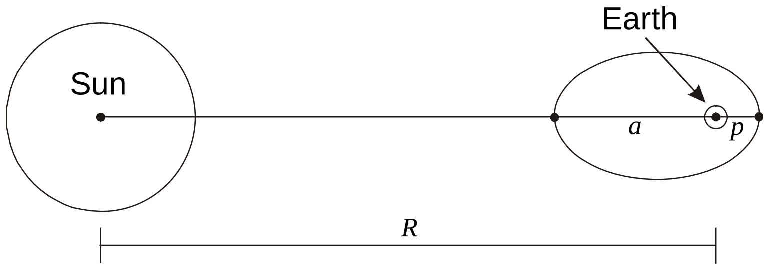
$$R = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3},$$

or 26.6 thousand kilometers (here *M* is the mass of the Earth).

2. Problem. The artificial satellite is rotating around the Earth along an elliptical orbit lying in the ecliptic plane. When it is in perigee point, the distances from Earth to the satellite and to Moon are the same. Estimate the maximum possible eccentricity of the satellite's orbit. Please don't consider the gravitation of the Moon.

2. Solution. Possible value of orbit eccentricity is restricted by tidal influence of the Sun (the influence of Moon is disregarded). If satellite comes close to the inner Lagrange point of the system Sun-Earth, the orbit will be instable and the satellite will come to heliocentric orbit, being the artificial planet of the Solar System. Let us consider the boundary case when the apogee point of the satellite is situated between the Sun and the Earth. Since the orbit lie in the ecliptic plane, such situation takes place once in a year.

Let us designate the perigee and apogee distances of the orbit as *p* and *a*, the distance between Sun and Earth as *R*. Assume that the satellite in the apogee came to the inner Lagrange point. This point rotates around the Sun with the same angular velocity ω as the Earth. Let us write the motion equations for Lagrange point and the Earth:



$$\frac{GM}{(R-a)^2} - \frac{Gm}{a^2} = \omega^2 (R-a);$$
$$\frac{GM}{R^2} = \omega^2 R.$$

Taking into account that $a \ll R$, we write

$$\frac{GM}{(R-a)^2} = \frac{GM}{R^2} + \frac{2GMa}{R^3}.$$

Expressing the value of ω from Earth motion equation, we receive the relation:

$$\frac{3GMa}{R^3} = \frac{Gm}{a^2}.$$

From this we make the estimation of maximal apogee distance

$$a = R\left(\frac{m}{3M}\right)^{1/3} = 1.497 \text{ mln. km}$$

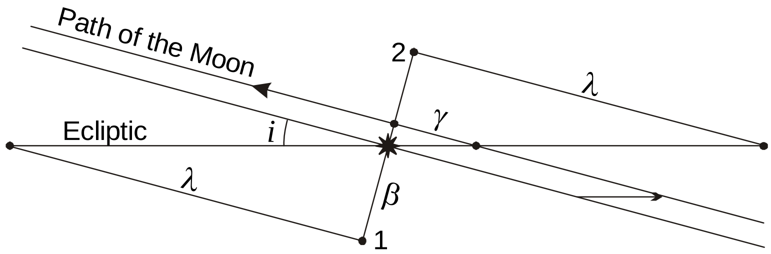
and orbit eccentricity (taking into account the value of p equal to 384 thousand kilometers):

$$e = \frac{a-p}{a+p} = 0.592.$$

3. Problem. The distant star is situated in the summer solstice point of the sky. During the passing of ascending node of lunar orbit near this star the lunar occultations of this star will be visible on the Earth every revolution of the Moon. How many occultations will this sequence contain? At what latitude and in what place of the sky will the first and the last occultation of the sequence be seen? The orbit of the Moon can be considered as circular.

3. Solution. If the Moon rotated around the Earth in the ecliptic plane, it would occult the star in the summer solstice point each revolution and this occultation would be seen in equatorial and tropical zones of our planet. But the inclination of lunar orbit to the ecliptic plane i is about 5.15° that is enough for the Moon to go above or below the star and to avoid occultation in most part of the cases. To occult the star, the Moon must be situated near one of two nodes of the orbit, where it crosses the ecliptic plane.

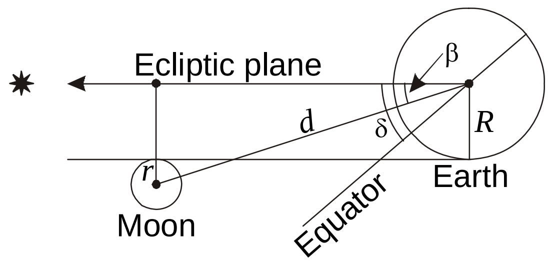
The sidereal period of lunar rotation T_S is equal to 27.321662 days and draconic period T_S (period of return to the same orbit node) is equal to 27.212220 days. Completing the revolution relatively stars, the Moon makes 1.00402 draconic revolutions. If the occultation occurs exactly in the ascending orbit node, next one will occur a month later slightly after the node crossing. The Moon will be a little bit higher and the region of occultation visibility will slightly shift northwards (see the figure).



The angular distance between the Moon and ascending node will be equal to:

$$\gamma = 360^\circ \cdot \frac{T_S - T_D}{T_D} = 1.448^\circ.$$

Shifting by this angle each sidereal month, line of nodes of lunar orbit will complete the revolution in 248.65 sidereal months or 18.6 years. This period will contain two epochs of occultation of the star near the ecliptic, near ascending and descending node, respectively. If the star is situated exactly at the ecliptic, these two sequences will be shifted by 9.3 years one from another. We have to find the duration of one sequence. Let us find the maximal distance between the star and the node for what the occultation is still possible.



Second figure shows the boundary situation, where the grazing occultation is observed just from one point on the Earth. The maximum angular distance between the Moon and ecliptic plane is equal to

$$\beta = \arcsin \frac{R + r}{d} = 1.209^\circ.$$

Here we assume the lunar orbit to be circular with radius d equal to 384.4 thousand kilometers, R and r are the radii of Earth and Moon. Returning to the first figure, we find maximal angular distance between the Moon and the orbit node:

$$\lambda = \beta \operatorname{ctg} i = 13.41^\circ.$$

The average number of occultation in one sequence is equal to:

$$N_A = \frac{2\lambda}{\gamma} = 18.52.$$

Real occultation number will be equal (with almost equal probabilities) to 18 or 19. The sequence duration will be equal to 17 or 18 sidereal months or 1.27 or 1.35 years.

In the case of ascending node the first occultation in a sequence will be near position 1 in the first figure, the same situation is shown in the second figure. The Moon will graze the star by the northern edge and this occultation will be observed in the southern hemisphere of the Earth. The Moon will be seen at the northern horizon (more exactly, at 5.15° westwards form the North point), the star declination is +23.4° and the occultation will be seen near Southern Polar Circle. Last occultation in a sequence (position 2 in the first figure) will be seen near North Polar Circle at 5.15° eastwards from the North point.

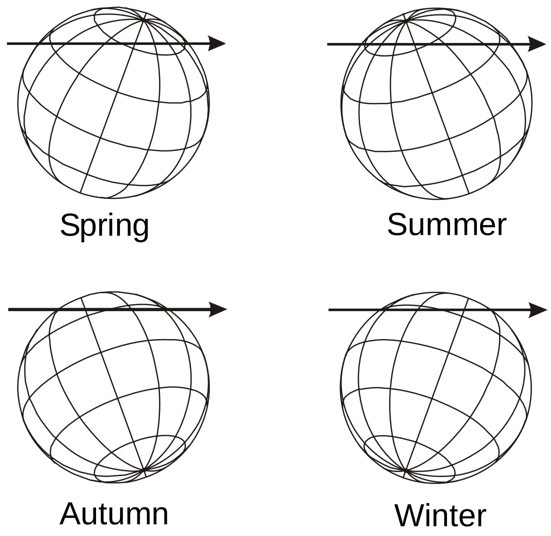
4. Problem. The path of total solar eclipse consistently passed the following cities: Oslo (Norway), Warsaw (Poland), Constanta (Romania), Ankara (Turkey), Baghdad (Iraq), Kerman (Iran) and Islamabad (Pakistan). What astronomical season (winter, spring, summer or autumn) the eclipse was observed in?

4. Solution. Table contains the coordinates of the cities listed in the problem.

City	Latitude, °	Longitude, °
Oslo	+59.9	+10.7
Warsaw	+52.2	+21.1
Constanta	+44.2	+28.6
Ankara	+39.9	+32.9
Baghdad	+33.3	+44.4
Kerman	+30.3	+57.1
Islamabad	+33.7	+73.1

As we can see, the shadow moved to the south-east for most part of time, and just in the end it moved to the east and even turned to the north-east. The shadow path is shown in the map.

Let us imagine how will the lunar shadow move on tropical and medium zone of the northern hemisphere in the middle of four seasons of the year (starting at the equinox or solstice). In the next figure we can see the Earth and shadow path at four seasons as it can be seen from the Sun (or from the Moon). The shadow moves from the left to the right (direction opposite to the motion of the Earth). Really the path motion direction is inclined to the ecliptic plane by the angle 5.15°, but it does not change the problem solution, since this angle is sufficiently less than the equator to ecliptic inclination (23.45°).

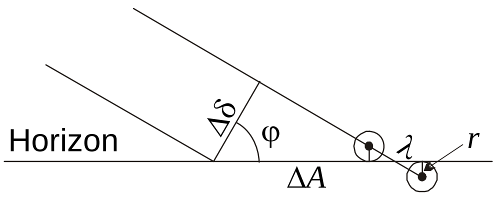


We can see that the shadow motion described in the problem can take place only during the astronomical autumn, between autumn equinox and winter solstice.

The problem has a real prototype: total solar eclipse in November, 19th, 1816, the map is above. The path of totality crossed the centers or surroundings of seven cities listed in the problem.

5. Problem. The astronomical azimuths of the rise and the following set of the planet in the city of Saint-Petersburg (latitude +60°) were equal to –90.0° and +90.4°. The duration of the planet disk set was equal to 3.2 seconds. What the planet is it and can it be said anything about the season, when it happened?

5. Solution. Values of azimuths show that the planet is situated near the celestial equator. Since the planets are always near the ecliptic, there were the surroundings of the vernal or autumn equinox points. The time between the rise and set of the planet is equal to 12 hours. The module of set azimuth is 0.4° higher than rise azimuth, thus the declination of the planet has increased during this time. Let us determine the value of this increase.



As we can see in the figure, change of azimuth module ΔA is related with change of declination $\Delta\delta$ and latitude of observation φ (equal to 60°) by the relation:

$$\Delta\delta = \Delta A \cos\varphi = +0.2°.$$

Account of atmospheric refraction will not change this result, since the refraction changes the value of azimuth but does not change their difference (ΔA) near the celestial equator. The planet moves along the ecliptic, it is true always except the planets' stationary moments, but their angular velocity is very small that time. Near the equinox points the ecliptic creates the angle $\varepsilon = 23.4°$ to the celestial parallels and equator. The total angular motion of the planet during 12 hours is equal to

$$\gamma = \frac{\Delta\delta}{\sin\varepsilon} = \Delta A \frac{\cos\varphi}{\sin\varepsilon} = 0.5°.$$

Finally, the angular velocity of the planet is equal to 1 per day. It is equal to visible angular velocity of the Sun. None of the planets can reach this angular velocity by the backward motion, and only two planets can move in the straight direction: Mercury and Venus, during their epoch of maximum elongation from the Sun.

To determine what is the planet, we determine its angular size. The duration of the planet set was equal to 3.2 seconds. Being near the equator, the planet shifted relatively observer on the Earth by the angle:

$$\lambda = 15'' \Delta t = 48''.$$

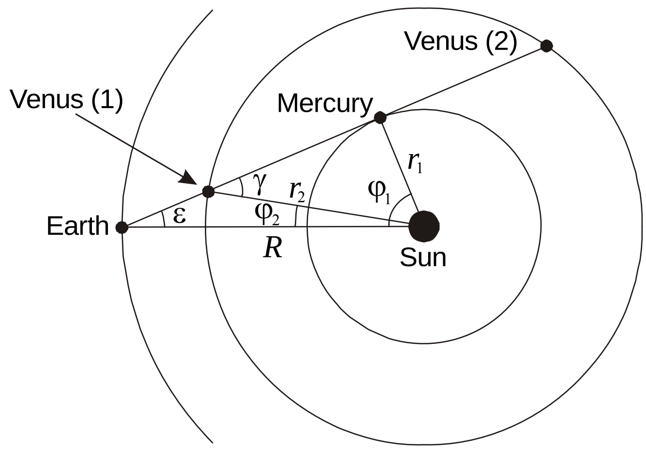
Angular diameter of the planet is equal to:

$$d = 2r = \lambda \cos\varphi = 24''.$$

This value is equal to angular diameter of Venus during the epoch of maximum elongation (the angular size of Mercury during this epoch is not larger than 8″). Venus is at 47° from the Sun and crossing the vernal equinox point by the straight direction. If it is maximum eastern elongation, than the picture is observed in early February, if it is maximum western elongation, than it is early May.

6. Problem. Being in the maximum eastern elongation point, Mercury came to the conjunction with Venus. The angular diameter of Mercury was more than 5 times less than the angular diameter of Venus. What planet will be the first to come to the inferior conjunction with Sun? What time will it be earlier than another planet? The orbits of Mercury, Venus and Earth can be considered as circular.

6. Solution. Figure shows the positions of Mercury, Venus and Earth in the moment described in the problem.



Being in conjunction with Mercury, Venus can be in two points of the orbit signed by the digits 1 and 2 in the figure. Angular diameter of Mercury during the maximum elongation epoch is about 8″, thus the angular diameter of Venus exceeds 40″. It can take place only if the Venus is in the position 1, close to Earth. Let us designate the orbit radii of Mercury, Venus and Earth as r_1, r_2, and R, respectively, and determine the heliocentric longitude difference of Earth and each of inner planets φ_1 и φ_2. It is easy to do for the Mercury:

$$\varphi_1 = \arccos\frac{r_1}{R} = 67.2^\circ.$$

The difference for Venus can be calculated from the triangle Sun-Venus-Earth with account of adjacent angle properties:

$$\varphi_2 = \gamma - \varepsilon = \arcsin\frac{r_1}{r_2} - \arcsin\frac{r_1}{R} = 9.6^\circ.$$

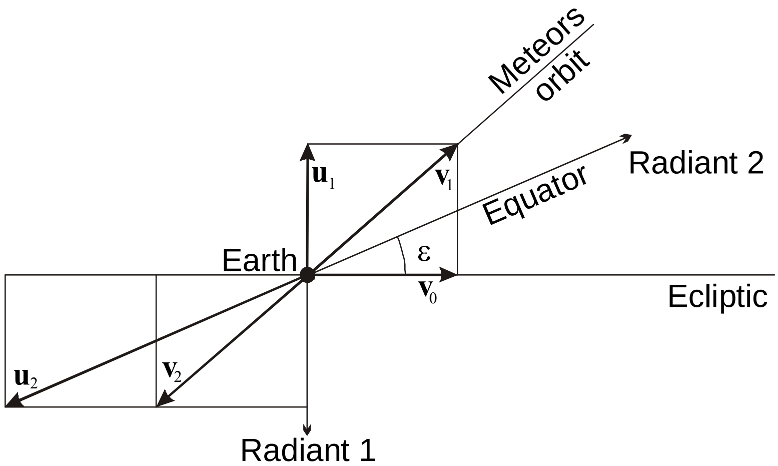
The time until the inferior conjunction of inner planet is equal to:

$$T_{1,2} = \frac{\varphi_{1,2}}{360^\circ} \cdot S_{1,2},$$

where S is the synodic period of inner planet, that is equal to 115.9 days for Mercury and 583.9 days for Venus. Substituting the numerical data, we obtain that inferior conjunction of Mercury will occur in 21.6 days and inferior conjunction of Venus will occur in 15.6 days. Finally, Venus will come to inferior conjunction in 6 days earlier than Mercury.

7. Problem. Two meteor streams are moving around the Sun by the same orbit, but in opposite directions. One moment both streams meet each other and the Earth. In the same moment two meteor showers are visible on the Earth, their radiant points have the coordinates $\alpha = 6^h$, $\delta = -66.6°$ and $\alpha = 18^h$, $\delta = 0°$. Please find the eccentricity of the meteors' orbit. What date did the streams meet the Earth? The orbit of Earth can be considered a circular one.

7. Solution. Since two meteor streams move by the same orbit in different directions, they will have equal velocities with opposite directions in the moment of Earth flyby. The radiant shows the directions opposite to geocentric velocity, that's why the radiants of two showers are not situated in opposite points of the sky.



Let us designate the vector of Earth's velocity as $\mathbf{v}_0$, the vectors of heliocentric velocities of meteor streams as $\mathbf{v}_1$ and $\mathbf{v}_2$, and their geocentric velocities as $\mathbf{u}_1$ and $\mathbf{u}_2$. For these velocities we can write the relations:

$$\mathbf{u}_1 = \mathbf{v}_1 - \mathbf{v}_0,$$
$$\mathbf{u}_2 = \mathbf{v}_2 - \mathbf{v}_0 = -\mathbf{v}_1 - \mathbf{v}_0.$$

Adding them to each other,

$$\mathbf{u}_1 + \mathbf{u}_2 = -2\mathbf{v}_0.$$

The radiant of first shower is situated in the South Ecliptic pole, thus, the figure plane containing the vector $\mathbf{u}_1$ is perpendicular to the ecliptic plane crossing it by the line containing the vector $\mathbf{v}_0$. Both vectors are perpendicular to the radius-vector of Earth, directed from the center of the Sun, and the whole figure plane including heliocentric meteor velocities is perpendicular to the line Sun-Earth. Thus, during the flyby the meteors were in the perihelion or aphelion of their orbit if this orbit is not circular.

During the event Earth is moving towards the ecliptic point that is one of two cross points of ecliptic and major circle of picture plane. The last one is the declination circle containing all sky points with right ascension values equal to 6^h and 18^h (it is easy to see basing on the radiants coordinates). Thus, these two points are the solstice points. As we can see in the last equation, the projections of the vectors $\mathbf{u}_2$ and $\mathbf{v}_0$ on the ecliptic have different signs (since the vector $\mathbf{u}_1$ is perpendicular to the ecliptic). Taking into account the coordinates of the second radiant we resume that the motion of Earth is directed to the winter solstice point. Such direction takes place during the vernal equinox, March, 21st. It is the answer to the second question of the problem.

The angle between the directions to the second radiant (celestial equator) and Earth velocity direction ε is equal to 23.4°. Projecting the vector equation for the sum of velocities $\mathbf{u}_1$ и $\mathbf{u}_2$ on two coordinate axes, we obtain:

$$u_1 = u_2 \sin\varepsilon,$$
$$2v_0 = u_2 \cos\varepsilon$$

and then

$$u_1 = 2v_0 \,\mathrm{tg}\,\varepsilon.$$

Heliocentric meteor velocity during the meeting event is equal to

$$v_1 = \sqrt{u_1^2 + v_0^2} = v_0\sqrt{1 + 4tg^2\varepsilon} = 39.4 \text{ km/s}.$$

This velocity exceeds the circular velocity v_0 but is less than parabolic velocity. Thus, the orbit of the meteors is elliptical and they are crossing the perihelion. Denoting the distance between Sun and Earth as r, we express the perihelion velocity:

$$v_1 = \sqrt{\frac{GM}{a}\cdot\frac{1+e}{1-e}} = \sqrt{\frac{GM}{r}(1+e)} = v_0\sqrt{1+e}.$$

Here a is the large semi-axis of the meteor orbit, e is its eccentricity and M is the solar mass. Finally,

$$e = 4\,\mathrm{tg}^2\varepsilon = 0.75.$$

8. Problem. While using the Giant Optical Cosmic telescope with ultra-high resolution astronomers of the future succeeded to see the disk of Betelgeuse (α Orionis). What object – Betelgeuse or Venus – will have higher surface brightness (the brightness of angular square unit)? What will the ratio of surface brightness of these objects be?

8. Решение. To solve the problem, we must remember that in the absorption-free case the surface brightness of the object does not depend on the distance. The total brightness is back proportional to the square of distance, but the same is the visible square of the object. For the self-emitting stars in the black body assumption the surface brightness is the function of surface temperature, increasing proportionally to its fourth degree. Betelgeuse is the red supergiant of spectral type M2 with the surface temperature about 3000 K, twice less than the solar one. Thus, surface brightness of Betelgeuse is about 16 times less than the one of the Sun.

Venus does not emit in the visible part of spectrum, but reflects the solar radiation. The ratio of surface brightness of Venus (center of the disk near superior conjunction) and the Sun is equal to

$$\frac{j_V}{j_0} = A\frac{r^2}{R^2} = \frac{1}{37300}.$$

Here A is the albedo of Venus, r is the radius of Sun and R is the orbit radius of Venus. Finally, surface brightness of Betelgeuse is about 2330 times higher than the one of Venus.

9. Problem. The globular stellar cluster has the magnitude equal to 4.5^m and angular diameter equal to 25′. The distance to the cluster is equal to 3 kpc. Having assumed that the cluster consists of the stars, which are like the Sun and their density is the same over the whole ball volume, so estimate the brightness of the night sky of the planet rotating around the star in the center of cluster. Compare it with the moonlit sky on the Earth. The absorption of light in the interstellar medium and planet's atmosphere can be ignored.

9. Solution. Expressing the angular diameter of stellar cluster in radians multiplying it to the distance and dividing by 2, we obtain the cluster radius r_0 equal to 10.9 pc. The cluster volume is equal to

$$V = \frac{4}{3}\pi r_0^3 = 5.42\cdot 10^3\,\text{pc}^3.$$

Absolute magnitude of the Sun M_0 is equal to $+4.7^m$. At the distance d equal to 3 kpc, Sun would be seen as the star with magnitude

$$m_0 = M_0 - 5 + 5\lg r = 17.1.$$

Denoting the total cluster magnitude as m, we express the number of stars in the cluster:

$$N = 10^{0.4(m_0 - m)} = 1.095\cdot 10^5$$

and the stars volume density:

$$n = \frac{N}{V} = 20.2\ \text{pc}^{-3}$$

In the night sky of the planet inside the cluster the stars of a half of the ball is seen, and they fill the sky uniformly. Let us designate the illumination from one star with magnitude 0^m as J and calculate the illumination from all stars in a thin semi-spherical shell with radius r and thickness Δr. The number of the stars in a shell is equal to

$$N_r = 2\pi r^2 \Delta r n\ .$$

The magnitude and illumination from each star are equal to

$$m = M_0 - 5 + 5\lg r,$$

$$j_r = \frac{J\cdot 10^{2-0.4M_0}}{r^2}.$$

Here r is designated in parsecs. The illumination from all stars in a shell is equal to

$$J_r = 2\pi nJ\cdot 10^{2-0.4M_0}\cdot \Delta r.$$

We see that this value is proportional to the thickness of the shell and does not depend on its radius. Performing the whole semi-sphere as the composition of such shells, we obtain the expression of the total illumination of planet night sky:

$$J_T = 2\pi nJ\cdot 10^{2-0.4M_0}\cdot r_0 = J\cdot 1.83\cdot 10^3.$$

The total magnitude is equal to:

$$m_T = -2.5\lg\frac{J_T}{J} = -8.2.$$

The sky is sufficiently brighter than the moonless sky on the Earth, and the objects outside the clusters will make just a small contribution to this value. But the moonlit night on the Earth (the Moon magnitude is equal to -12.7^m) is 63 times brighter than the sky of the planet in the center of the globular cluster.

10. Problem. The radius of the Galaxy is equal to 15 kpc, the thickness of its disk being many times less. The mass of the galaxy is equal to 10^{11} solar masses and it is distributed uniformly in the volume of the galaxy. Two stars are rotating around the center of the galaxy in the same direction by the circular orbits with radii equal to 5 kpc and 10 kpc. Please find the synodic period of the first star while observing from the vicinity of the second star.

10. Solution. The mass density inside the galaxy is equal to

$$\rho = \frac{M}{\pi R^2 d}.$$

Here M, R and d are the mass, radius and thickness of the galaxy. We assume that the motion of star by the circular orbit with radius r is influence only by the part of the galaxy inside the cylinder with the same radius. Mass of this part is equal to

$$m(r) = \pi r^2 d\rho = M \frac{r^2}{R^2}.$$

The angular velocity of the star's rotation around the center of the galaxy is equal to

$$\omega(r) = \sqrt{\frac{Gm(r)}{r^3}} = \sqrt{\frac{GM}{R^3} \cdot \frac{R}{r}} = \omega_0 \sqrt{\frac{R}{r}}.$$

Here ω_0 is the angular velocity of the rotation of galaxy edge. Note that such dependency is weaker than the one by Kepler law (with the degree –3/2). Synodic period of the star with orbit radius r_1 observed from the star with radius r_2 is equal to

$$S = \frac{2\pi}{\omega(r_1) - \omega(r_2)} = 2\pi \sqrt{\frac{R^3}{GM}} \cdot \frac{1}{\sqrt{R/r_1} - \sqrt{R/r_2}}$$

Substituting the numeric values, we obtain 1.07 billions of years.